Causality in nonlocal gravity*

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Abstract

The definition of quantum gravity is hindered by the difficulty of reconciling the requirements of renormalizability and unitarity. Recently it has also been pointed out that some higher-derivative which may appear in modifications of Einstein-Hilbert gravity terms are associated with a violation of causality at scales larger than the Planck length [1]. This can be avoided by adding an infinite tower of massive higher spin particles, which are in fact expected in a weakly coupled string theory. On the other hand this argument poses a crucial test for the causality of quantum gravity theories containing only conventional particles with spin not greater than 2.

We review several results about a class of weakly nonlocal purely gravitational (or coupled to matter) theories that are compatible with perturbative unitarity and finiteness at quantum level. In particular,we argue the requirement of causality can be satisfied avoiding some higher derivative terms.This result is justified computing Shapiro's time delay in terms of tree-level scattering amplitudes for nonlocal gravity models with and without matter. We also show how generic nonlocal gravity theories consistent with causality can be obtained by a field redefinition from standard local theories.

1. Introduction

The major challenge of quantum gravity lies in the difficulty of reconciling renormalizability and perturbative unitarity. In fact, the Einstein-Hilbert action is not power counting renormalizable, but, if we introduce the infinite number of counterterms generated by the renormalization procedure, it is

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perturbatively unitary. On the other hand, it is possible to build higher-derivatives theories of quantum gravity that are renormalizable with finitely many counterterms, but are non-unitary. This is the case for example of the celebrated Stelle theory [2]. In the literature a number of possible solutions to this puzzle have been discussed. Among them in recent years nonlocal models received a great deal of attention both for their capability to produce an interesting phenomenology and for their remarkable properties upon quantization. In particular, a class of weakly nonlocal gravitational theories has been proven to be super-renormalizable (or finite) and perturbatively unitary[3, 4, 5, 6, 7, 8]. These theories have also been studied in connection to cosmological backgrounds and black hole solutions [9, 10, 11, 12, 13].

However the main concern about nonlocal theories is surely causality, whose investigation has only recently been addressed in a systematic way. In a classical theory the problem is closely related to a mathematically sound formulation of the initial value problem whereas from the quantum perspective nonlocality implies a new formulation of the Bogoliubov causality condition for local interactions. Remarkably, for quasi-local interactions where the nonlocality shows up only at scales smaller than the nonlocality scale ℓ_Λ, non causal effects remain confined within the scale ℓ_Λ [14, 15]. This has also led to the idea that for asymptotically free nonlocal theories, this violation of microcausality may actually be undetectable [16].

In this note, we consider another notion of causality introduced by Gao and Wald [17], according to which it is impossible to send signals faster than what is allowed by the asymptotic causal structure of the spacetime. Its violation has been recently discussed by Camanho, Edelstein, Maldacena, and Zhiboedov [1], in particular in connection to Shapiro's time delay, which is one of the classical tests of general relativity (GR). Light propagating near a compact object should suffer a time delay compared with the same propagation in flat spacetime. Therefore, if we get a negative time delay, or actually a time advancement, we have a causality violation. Exploiting the relation between the Shapiro time delay and the scattering amplitudes for gravitating particles in the eikonal approximation, the authors of [1] have proven that these causality problems are produced only by the form of the on-shell three-point functions of the theory. Therefore the most general higher derivative gravity theory giving rise to a causality violation is at most cubic in the Riemann tensor. In particular their analysis applies to a high energy scattering process in which gravity is still weakly coupled. This can be achieved if the impact parameter b can be chosen such that

$$\ell_{\rm P} \ll b \ll \ell_\Lambda \,, \tag{1}$$

where $\ell_{\rm P}$ the Planck scale, and ℓ_Λ the non locality scale. In such cases the loss of causality can be evaded by adding massive higher spin particles with spin $J > 2$ and mass $m^2 \sim \ell_\Lambda^{-2}$.

In the following we want to argue that the terms responsible for this causality violation do not need to show up in a nonlocal theory of quantum gravity of the kind studied in [3, 4, 5, 6, 7, 8]. So these theories turn out to be consistent even without the introduction of an infinite tower of massive higher spin fields. First, in section 2. we review some essential features of the class of theories under consideration, in particular their renormalization properties and perturbative unitarity. Then, in section 3. we review results about scattering amplitudes in weakly nonlocal theories pointing out the crucial role played by a theorem relating tree-level amplitudes in theories related by field redefinitions [22, 23]. In section 4. we finally report about the results of [24], where the problem of causality has been addressed.

2. Weakly nonlocal gravity

We investigate the class of theories defined by the action

$$S_{\mathrm{g}} = \frac{2}{\kappa_D^2} \int d^D x \sqrt{-g}\left[R + G_{\mu\nu}\gamma(\Box)R^{\mu\nu} + V(\mathcal{R})\right] , \tag{2}$$

where $\kappa_D^2 = 32\pi G$. Given the non-locality scale $\sigma \equiv \ell_\Lambda^2$, the form factor $\gamma(\Box)$ is defined by

$$\gamma(\Box) = \frac{e^{H(\sigma\Box)} - 1}{\Box} , \tag{3}$$

where the function $\exp H(z)$ is asymptotically polynomial in a conical region C around the real axis, namely

$$|\exp H(z)| \to |z|^{\gamma+\mathrm{N}+1} \quad \text{for} \quad |z| \to +\infty, \tag{4}$$

with N an integer defined in terms of the spacetime dimension D so that $2N+4 = D$ (if D is even) or $2N+4 = D+1$ (if D is odd). This condition is necessary to avoid the appearance of nonlocal counterterms in the UV regime. An example due to Tomboulis [4] is

$$H_T(z) = \frac{1}{2}\left[\Gamma\left(0, p(z)^2\right) + \gamma_E + \log\left(p(z)^2\right)\right] , \tag{5}$$

where $p(z)$ is a polynomial of degree γ+N+1, $\Gamma(a,z)$ the incomplete Gamma function and γ_E the Euler-Mascheroni constant. The local potential $V(\mathcal{R})$ is at least cubic in the curvature, namely $V \sim O(\mathcal{R}^3)$, but quadratic in the Ricci tensor, and is taken to contain at most $2\gamma + 2N + 4$ derivatives.

These choices are motivated by inspection of quantum divergence in the UV regime. In fact the graviton propagator scales as $k^{-(2\gamma+2N+4)}$ and the vertices contain terms whose leading behavior is just the inverse. This determines the upper bound on the superficial degree of divergence of any graph G, $\omega(G) \equiv DL + (V-I)(2\gamma+2N+4)$. We find in a spacetime of even or odd dimension respectively:

$$\omega(G)_{even} = D_{even} - 2\gamma(L-1), \quad \omega(G)_{odd} = D_{odd} - (2\gamma+1)(L-1) . \tag{6}$$

Thus, if $\gamma > D_{even}/2$ or $\gamma > (D_{odd} - 1)/2$, only 1-loop divergences survive. Therefore, the theory is super-renormalizable and only a finite number of operators of mass dimension up to M^D has to be included in the action for renormalization in even dimensions. In odd dimensions, due to dimensional reasons, there are no divergences at one loop and the theory is automatically finite. The freedom in the choice of the potential $V(\mathcal{R})$ can be used to "kill" the one-loop divergences in even dimensions. For example, in 4 dimensions it is possible to prove that the two quartic killer operators

$$s_1 \, R^2 \, \Box^{\gamma-2} R^2 + s_2 \, R_{\mu\nu} R^{\mu\nu} \, \Box^{\gamma-2} R_{\rho\sigma} R^{\rho\sigma} \tag{7}$$

give contributions to the beta functions of the couplings for R^2 and $R^2_{\mu\nu}$ which are linear in their front coefficients s_1 and s_2 so that finiteness can be achieved by choosing them so that $\beta_{R^2} = \beta_{R^2_{\mu\nu}} = 0$. The crucial point for the following is that the killer terms should be in general at least quadratic in the Ricci tensor.

One can easily find from the kinetic term the two-point function in the harmonic gauge ($\partial^\mu h_{\mu\nu} = 0$)

$$\mathcal{O}^{-1} \sim \frac{1}{k^2 e^{H(k^2/\Lambda^2)}} \left(P^{(2)} - \frac{P^{(0)}}{D-2} \right) , \tag{8}$$

where $P^{(0)}$ and $P^{(2)}$ are the usual spin 2 and spin 0 projectors. Therefore perturbative unitarity together with the absence of gauge invariant poles other than the graviton pole requires that $\exp H(z)$ be real and positive on the real axis and without zeros on the whole complex plane $|z| < +\infty$. This choice however implies a subtlety related to the fact that amplitudes are well-defined as integrals along certain loop integration contours and in a certain regime of external momenta, which is typically the Euclidean one. The vertices we have defined in order to achieve UV finiteness must decrease sufficiently fast along some directions in the complex plane, namely the ones corresponding to Euclidean momenta. However for the non-polynomial entire functions this necessarily implies a fast growth in other directions in the complex plane, thus generally preventing the usual Wick rotation. This could generically point at a violation of perturbative unitarity. However, the theories considered in this note turn out to be unitary at perturbative level to all perturbative orders in the loop expansion as rigorously and extensively proved in [18] and more recently in [19, 20, 21] The proof is based on an analytic continuation of the external particles' energies from imaginary to real values. It turns out that the Landau singularities, and the discontinuities of the amplitudes are the same of a local theory at any perturbative order in the loop expansion. This is a consequence of the classical spectrum of the theory that is the same of the local theory. Therefore, the Cutkosky cutting rules are the same of the local theory. Finally, there is no contribution of cut diagrams corresponding to anomalous thresholds to the imaginary part of the scattering amplitudes as proved in [19, 21]. Indeed, if a diagram is cut in less then two or more then two parts the contribution to the discontinuities vanishes as a consequence of the energy momentum conservation.

3. Scattering amplitudes in higher derivative gravity theories

We here review some results about scattering amplitudes in higher derivative gravity theories, in particular four-graviton tree-level ones in the case when all higher derivative terms are at least quadratic in the Ricci tensor. For the sake of clarity, we first consider an action

$$\begin{aligned} S_g &= -2\kappa_D^{-2}\int d^Dx\,\sqrt{-g}\Big(R + R\gamma_0 R + R_{\mu\nu}\gamma_2 R^{\mu\nu} + \\ &\quad (R_{\mu\nu\rho\sigma}\gamma_4 R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\gamma_4 R^{\mu\nu} + R\gamma_4 R)\Big)\,, \end{aligned} \tag{9}$$

where γ_0, γ_2, γ_4 are generic functions of $\sigma\Box$. As observed in [22], in such cases the computation can be addressed by standard Feynman diagram techniques due to a number of simplifications. First of all, we note the last term is the famous Gauss-Bonnet density (GB), which is topological in four dimensions whereas for generic higher dimensions it gives rise to vertices only. Furthermore, as the process involves only three-graviton vertices with two gravitons on-shell and one off-shell and a four-graviton vertex with all external legs on-shell, we can make full use of the linearized vacuum equation of motion for the physical field $h_{\mu\nu}$ in the harmonic gauge, i.e. $\Box h_{\mu\nu} = 0$. Actually, we can choose polarizations satisfying the conditions $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$ all along the computation, which greatly simplifies the algebra. In fact, these conditions imply that all the scalar operators are vanishing on-shell at linear order in $h_{\mu\nu}$, including the scalar curvature $R^{(1)}$ and the root of metric determinant $\sqrt{-g}^{\,(1)}$. One can further show that $R^{(1)}_{\mu\nu} = 0$ due to the linearized EOM. We can express all the amplitudes in terms of the Mandelstam variables $s = 4E^2$, $t = -2E^2(1-\cos\theta)$ and $u = -2E^2(1+\cos\theta)$, with E the energy and θ the scattering angle in the center-of-mass reference frame.

In the case where γ_0, γ_2, γ_4 are constants, for gravitons with positive helicity, one finds for $D = 4, 5, 6$

$$\mathcal{A}^{D=4}(++,++) = i\frac{1}{\kappa_4^2}\frac{4E^2}{\sin^2\theta}\,, \tag{10}$$

$$\begin{aligned} &\mathcal{A}^{D=5}(++,++) = \\ &\quad -i\frac{2}{\kappa_5^2}\left\{\frac{16E^6\gamma_4^2\left[1+8E^2(3\gamma_0+\gamma_2)\right]}{(1-4E^2\gamma_2)\left[3+4E^2(16\gamma_0+5\gamma_2)\right]} - 2E^2\frac{1}{\sin^2\theta}\right\}, \end{aligned} \tag{11}$$

$$\mathcal{A}^{D=6}(++,++) = \\ -i\frac{2}{\kappa_6^2}\left\{\frac{8E^6\gamma_4^2\left[1+8E^2(3\gamma_0+\gamma_2)\right]}{(1-4E^2\gamma_2)\left[1+2E^2(10\gamma_0+3\gamma_2)\right]} - 2E^2\frac{1}{\sin^2\theta}\right\}. \tag{12}$$

Remarkably, in 4 dimensions, where γ_4 cannot enter the amplitude because of the Gauss-Bonnet theorem, the result coincides with the one expected in Einstein theory by dimensional analysis and symmetry arguments . In particular no term scaling as E^4 in the UV shows up as it would be natural to expect in a four derivative theory. This is the result of non-trivial cancellations between the massive poles in the propagator and the three-graviton vertices and between the contact and exchange diagrams. This result can be also understood as the one consistent with the natural expectation in the limit where the Einstein term can be dropped out only leaving the scaleless quadratic terms. They would be expected to naively give amplitudes $\sim E^4$, but this cannot happen because the graviton field is dimensionless and there is no other scale. So the amplitude is actually expected to vanish. In $D>4$ γ_4 enters the amplitude, but only quadratically whereas the expected linear contribution is absent due to a cancellation between the contact diagram with a vertex from the Gauss-Bonnet term and the exchange diagrams with two different vertices (one from GB, the second one from standard terms R, R^2 or $R^2_{\mu\nu}$). Whereas in the ultraviolet regime the amplitude scales as E^4, in the infrared one finds arbitrary powers of E^2 associated with the massive poles in the propagators which cannot cancel with the three-graviton vertices of the Gauss-Bonnet density.

For weakly nonlocal gravity the amplitude can be also performed straightforwardly if $\gamma_4(\Box)=0$. Infact, as on-shell $\mathbf{R}\sim O(h^2)$ and $\mathbf{Ric}\sim O(h^2)$ (whereas $\mathbf{Riem}\sim O(h)$), the form factors are spectators in the expansion in the number of gravitons and many results for the Stelle gravity apply to the general nonlocal theory. For the three exchange diagrams and the contact one, we find

$$\mathcal{A}_s(++,++) = \\ -2\kappa_4^{-2}\left(-\frac{9}{8}\frac{t(s+t)}{s}+\frac{9}{32}\gamma_2(s)\left(s^2+(s+2t)^2\right)+\frac{9}{8}s^2\gamma_0(s)\right), \tag{13}$$

$$\mathcal{A}_t(++,++) = -2\kappa_4^{-2}\left(-\frac{1}{8}\frac{\left(s^3-5s^2t-st^2+t^3\right)(s+t)^2}{s^3t}\right. \\ \left. +\frac{1}{16}\gamma_2(t)\frac{\left(2s^4-10s^3t-s^2t^2+4st^3+t^4\right)(s+t)^2}{s^4}+\frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4}\right), \tag{14}$$

$$\mathcal{A}_u(++,++) = -2\kappa_4^{-2}\left(-\frac{1}{8}\frac{\left(s^3-5s^2u-su^2+u^3\right)(s+u)^2}{s^3u}+\right.$$

$$\left.\frac{1}{16}\gamma_2(u)\frac{\left(2s^4-10s^3u-s^2u^2+4su^3+u^4\right)(s+u)^2}{s^4}+\frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\right), \tag{15}$$

$$\mathcal{A}_{\text{contact}}(++,++) =$$

$$-2\kappa_4^{-2}\left(-\frac{1}{4}\frac{s^4+s^3t-2st^3-t^4}{s^3}-\frac{9}{32}\gamma_2(s)\left(s^2+(s+2t)^2\right)-\frac{9}{8}s^2\gamma_0(s)\right.$$

$$-\frac{1}{16}\gamma_2(t)\frac{\left(2s^4-10s^3t-s^2t^2+4st^3+t^4\right)(s+t)^2}{s^4}-\frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4}-$$

$$\left.\frac{1}{16}\gamma_2(u)\frac{\left(2s^4-10s^3u-s^2u^2+4su^3+u^4\right)(s+u)^2}{s^4}-\frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\right), \tag{16}$$

where the possible poles associated with γ_0 and γ_2 cancel separetely in each channel. Once again, the amplitude

$$\mathcal{A}(++,++) = \mathcal{A}_s(++,++) + \mathcal{A}_t(++,++)+$$

$$\mathcal{A}_u(++,++) + \mathcal{A}_{\text{contact}}(++,++) = \mathcal{A}(++,++)_{\text{EH}}\,, \tag{17}$$

coincides with the one in Einstein-Hilbert theory.

These results, which may look somewhat surprising, find actually a very natural explanation in terms of a field redefinition theorem [22, 23] that allows to map nonlocal field theories to local ones at tree-level.

In particular, let us consider two general weakly nonlocal actions, namely $S'(g,\Phi_a)$ and $S(g',\Phi'_a)$, respectively defined in terms of the fields g,Φ_a and g',Φ'_a, where g is the metric and Φ_a a set of matter or gauge fields, and such that

$$S'(g,\Phi_a) = S(g,\Phi_a) + E_i^g(g,\Phi_a)F_{ij}^g(g,\Phi_a)E_j^g(g,\Phi_a)$$

$$+E_a^\Phi(g,\Phi_c)F_{ab}^\Phi(g,\Phi_c)E_b^\Phi(g,\Phi_c)\,, \tag{18}$$

where F^g and F^Φ can contain derivative operators or weakly nonlocal operators of the covariant $\Box$ operator, and

$$E_i^g = \frac{\delta S}{\delta g_i}\,, \quad E_a^\Phi = \frac{\delta S}{\delta \Phi_a} \tag{19}$$

are the EOM of the theory with action $S(g, \Phi_a)$. The statement of the the theorem is that there exists a field redefinition

$$
\begin{aligned}
g'_i &= g_i + \Delta^g_{ij} E^g_j\,, \qquad \Delta^g_{ij} = \Delta^g_{ji}, \\
\Phi'_a &= \Phi_a + \Delta^\Phi_{ab} E^\Phi_b\,, \qquad \Delta^\Phi_{ab} = \Delta^\Phi_{ba}\,,
\end{aligned} \tag{20}
$$

such that, perturbatively in $F^{g,\Phi}$, but to all orders in powers of $F^{g,\Phi}$, we have the equivalence

$$
S'(g, \Phi) = S(g', \Phi')\,. \tag{21}
$$

The indices i, a on fields we encode all Lorentz, group indices, and the spacetime dependence of the fields. Δ^g_{ij} (Δ^Φ_{ab}) could be a weakly nonlocal or quasi-polynomial operator acting linearly on the EOM E^g_j (E^Φ_a), and they are defined perturbatively in powers of the operators $F^{g,\Phi}$, namely

$$
\Delta^g_{ij} = F^g_{ij} + \ldots \quad \text{or} \qquad \Delta^\Phi_{ab} = F^\Phi_{ab} + \ldots\,. \tag{22}
$$

The claim above can be straightforwardly checked at the first order in the Taylor expansion for the functional $S(g', \Phi'_a)$

$$
\begin{aligned}
S(g', \Phi'_a) = S(g + \delta g, \Phi + \delta\Phi) &\approx S(g) + \frac{\delta S}{\delta g_i}\delta g_i + \frac{\delta S}{\delta \Phi_a}\delta\Phi_a \\
&= S(g) + E^g_i\, \delta g_i + E^\Phi_a\, \delta\Phi_a\,,
\end{aligned} \tag{23}
$$

which is consistent with the equivalence (21) if we assume the field redefinitions (20) with the chosen coefficients (22). The theorem states the equivalence of the two theories only perturbatively in $F^{g,\Phi}$, so that the two theories do not need to be equivalent in all aspects. For example $S'(g, \Phi)$ can have additional poles in the spectrum compared with $S(g', \Phi')$ and also the quantum behaviors of the the theories can be completely different. However, the theorem applies to all the n-points tree-level functions whose external legs are on the mass-shell shared by the two theories, and this explains the results found by direct computation for theories that are quadratic in both the Ricci and scalar curvature and lack a term quadratic in the Riemann tensor.

4. Shapiro's time delay

The results of the previous section can be nicely translated in the language of Shapiro's time delay. This can in fact be recovered from the scattering amplitudes in the so-called eikonal approximation, which resums a particular set of diagrams (horizontal ladders) in the deflectionless limit $t/s << 1$. s is large compared to the inverse of the nonlocality scale ℓ_Λ^{-2}, but still well below the Planck scale so that the theories we are considering are still weakly coupled. Under favorable circumstances the amplitude exponentiates in the impact parameter space [25, 26]

$$iA_{\rm eik} = 2s\int d^{D-2}\vec{b}\, e^{-i\vec{q}\cdot\vec{b}}\left[e^{i\delta(b,s)}-1\right], \tag{24}$$

where the phase is given by

$$\delta(b,s) = \frac{1}{2s}\int\frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}}e^{i\vec{q}\cdot\vec{b}}A_{\rm tree}(s,-\vec{q}^{\,2})\,. \tag{25}$$

Shapiro's time delay is then given by

$$\Delta t = 2\partial_E\delta(E,b)\,. \tag{26}$$

where E is the energy of the probe-particle.

In particular, for the action (9), with $\gamma_0 = \gamma_2 = 0$ and $\gamma_4 = \lambda_{GB}$ a constant, one finds for the four graviton amplitude in the Regge limit [27]

$$A_t = A_{t{\rm EH}} + A_{t{\rm GB}} \approx -\frac{8\pi G s^2}{t}(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4)$$

$$+\frac{\kappa_D^2\lambda_{\rm GB}s^2}{t}\left(k_2^\mu k_4^\nu\epsilon_{2\nu}^\rho\epsilon_{4\rho\mu}\epsilon_1\cdot\epsilon_3 + k_1^\mu k_3^\nu\epsilon_{1\nu}^\rho\epsilon_{3\rho\mu}\epsilon_2\cdot\epsilon_4\right),$$

where $\kappa_D^2 = 32\pi G$, the momenta of the four gravitons k_1, k_2, k_3, k_4 are all incoming, i.e. $\sum_{i=1}^4 k_i = 0$, and ϵ_i $(i = 1,\ldots,4)$ are the polarizations of the gravitons. Choosing the metric $ds^2 = -dudv + \sum_{i=1}^{D-2}\left(dx^i\right)^2$, we can evaluate this amplitude in the following momentum configuration

$$\begin{aligned} k_{1\mu} &= \left(k_u, \frac{\overrightarrow{q}^2}{16k_u}, \frac{\overrightarrow{q}}{2}\right), \quad k_{3\mu} = -\left(k_u, \frac{\overrightarrow{q}^2}{16k_u}, -\frac{\overrightarrow{q}}{2}\right) \\ k_{2\mu} &= \left(\frac{\overrightarrow{q}^2}{16k_v}, k_v, -\frac{\overrightarrow{q}}{2}\right), \quad k_{4\mu} = -\left(\frac{\overrightarrow{q}^2}{16k_v}, k_v, \frac{\overrightarrow{q}}{2}\right) \\ s &\simeq 4k_uk_v, \quad t \simeq -(\overrightarrow{q})^2, \end{aligned} \tag{27}$$

where we just kept the leading order in the t/s expansion, assuming $t/s \gg 1$. We also take the polarizations $\epsilon^{\mu\nu} = \epsilon^\mu\epsilon^\nu$, given by

$$\epsilon_1^\mu = \left(-\frac{\overrightarrow{q}\cdot\overrightarrow{e}_1}{2k_u}, 0, \overrightarrow{e}_1\right), \quad \epsilon_3^\mu = \left(\frac{\overrightarrow{q}\cdot\overrightarrow{e}_3}{2k_u}, 0, \overrightarrow{e}_3\right) \tag{28}$$

$$\epsilon_2^\mu = \left(0, \frac{\overrightarrow{q}\cdot\overrightarrow{e}_2}{2k_v}, \overrightarrow{e}_2\right), \quad \epsilon_4^\mu = \left(0, -\frac{\overrightarrow{q}\cdot\overrightarrow{e}_4}{2k_v}, \overrightarrow{e}_4\right). \tag{29}$$

Choosing $e_1 = e_3$ and $e_2 = e_4$ we can compute the phase (25) for the Einstein-Hilbert term

$$\delta_{\rm g}(b,s) = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{Gs}{b^{D-4}} \, (e_1 \cdot e_3)(e_2 \cdot e_4) \, , \tag{30}$$

and for the Gauss-Bonnet term

$$\begin{aligned} \delta_{\rm GB}(b,s) &= 4\lambda_{\rm GB} \left(\left(e_1^{ij} e_1^{ij} \right) e_2^{ij} e_2^{ik} + \left(e_2^{ij} e_2^{ij} \right) e_1^{ij} e_1^{ik} \right) \partial_{b_i} \partial_{b_j} \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{Gs}{b^{D-4}} \\ &= -4\lambda_{\rm GB} \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{Gs}{b^{D-2}} \left[2(e_1 \cdot e_1)(e_2 \cdot e_2) \right. \\ & \quad \left. -(D-2)(n \cdot e_1)^2 - (D-2)(n \cdot e_2)^2 \right] \, , \end{aligned} \tag{31}$$

where $\vec{n} \equiv \vec{b}/b$. The total contribution to the phase is given by the sum of (30) and (31), namely

$$\delta_{\rm g-GB}(b,s) = \delta_{\rm g}(b,s) + \delta_{\rm GB}(b,s). \tag{32}$$

Finally, the Shapiro's time delay is

$$\begin{aligned} \Delta t_{\rm g-GB} &= \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{16EG}{b^{D-4}} (e_1 \cdot e_1)(e_2 \cdot e_2) \\ & \left[1 + \frac{4\lambda_{\rm GB}(D-2)(D-4)}{b^2} \left(\frac{(n \cdot e_1)^2}{e_1 \cdot e_1} + \frac{(n \cdot e_2)^2}{e_2 \cdot e_2} - \frac{2}{D-2} \right) \right] . \end{aligned} \tag{33}$$

We can see that if the impact factor b^2 becomes small, $b^2 < \lambda_{\rm GB}$, the third term in (33) can be bigger then the first two, depending on the sign of $\lambda_{\rm GB}$ and the polarizations. Therefore, we can have a time advance and causality is violated.

On the other hand, we saw in the previous sections that for theories that are quadratic in both the Ricci and scalar curvature and lack a term quadratic in the Riemann tensor, the tree-level amplitude exactly coincides with the Einstein-Hilbert one. The corresponding time delay is

$$\Delta t_{\rm g} = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{16EG}{b^{D-4}} \, (e_1 \cdot e_3)(e_2 \cdot e_4) \, , \tag{34}$$

and of course no time advancement is possible.

Actually, *any nonlocal theory that is tree-level equivalent by the field redefinition theorem to a causal local one is causal too. In other words, given a causal (possibly local) theory, the theorem provides an algorithm for constructing a full class of higher derivative (even non-local) causal theories.*

An explicit example of a nonlocal theory involving gravity, one gauge field, and a scalar field is the one given by the action

$$\mathcal{L} = \frac{2}{\kappa_D^2}\left[R + \left(G_{\mu\nu} - \frac{\kappa_D^2}{2}(T^A_{\mu\nu} + T^\phi_{\rho\sigma})\right) F_{\rm g}^{\mu\nu,\rho\sigma}\left(G_{\rho\sigma} - \frac{\kappa_D^2}{2}(T^A_{\rho\sigma} + T^\phi_{\rho\sigma})\right)\right]$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \nabla_\mu F^{\mu\nu}\, F^{\rm A}\, \nabla_\rho F^\rho{}_\nu$$
$$+\frac{1}{2}\phi(\Box - m^2)\phi + \phi(\Box - m^2)\, F^\phi\, (\Box - m^2)\phi\,, \tag{35}$$

where the analytic functions of the d'Alembertian operator $F^{\rm g}$, F^A, F^ϕ and the second rank tensors $T^A_{\mu\nu}$, $T^\phi_{\mu\nu}$ are defined as follows,

$$F_{\rm g}^{\mu\nu,\rho\sigma} \equiv \left(g^{\mu\rho}g^{\nu\sigma} - \frac{1}{2}g^{\mu\nu}g^{\rho\sigma}\right)\left(\frac{e^{H_{\rm g}(\Box)} - 1}{\Box}\right),$$
$$F^{\rm A} \equiv \frac{1}{2}\left(\frac{e^{H_A(\Box)} - 1}{\Box}\right),$$
$$F^\phi \equiv \frac{1}{2}\left(\frac{e^{H_\phi(\Box - m^2)} - 1}{\Box - m^2}\right),$$
$$T^A_{\mu\nu} \equiv F_{\mu\sigma}F^\sigma_\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$
$$T^\phi_{\mu\nu} \equiv \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial_\lambda\phi\partial^\lambda\phi + m^2\phi^2)\,. \tag{36}$$

$H_{\rm g}$, H_A and H_ϕ are form factors suitably chosen so that the theory is unitary and finite at quantum level in odd dimension (in particular in $D = 5$). In particular the theory (35) has the same spectrum of the equivalent local theory, namely the graviton, the photon, and the real scalar field (see [5, 7, 8] for more details) and by the field redefinition theorem all the tree-level n-point functions for the theory (35) are identical to the ones in local Einstein-Hilbert gravity coupled to the local Maxwell field, and a local scalar field. It is straightforward to prove that the theory above satisfies the field redefinition theorem, namely it is equivalent to Einstein's gravity minimally coupled to the electromagnetic field and scalar matter. Therefore, all the tree-level n-point functions for the theory (35) are identical to the ones that one can compute in local Einstein-Hilbert gravity couple to the local Maxwell field, and a local scalar field. In particular we can consider the elastic scattering of gravitons on massive scalars, and the photon-graviton scattering, whose amplitudes read

$$A(h,\phi;h,\phi)_{2;2} = 8\pi G\,\frac{(m^4 - su)^2}{t(s + m^2)(u + m^2)}\,,$$

$$A(h,\phi;h,\phi)_{-2;2} = 8\pi G\,\frac{m^4 t}{(s+m^2)(u+m^2)}\,, \tag{37}$$

$$A(h,A;h,A)_{1,2;1,2} = -8\pi G\,\frac{u^2}{t}\,,$$

$$A(h,A;h,A)_{1,-2;1,-2} = -8\pi G\,\frac{s^2}{t}\,, \tag{38}$$

plus the ones one can obtain by parity conjugation.

Taking the massless limit of (37), the helicity flip amplitude vanishes, while

$$A(h,\phi;h,\phi)_{2;2} = 8\pi G\,\frac{su}{t}\,. \tag{39}$$

All the above amplitudes in the eikonal limit $s \ll t$ simplify to:

$$-8\pi G\,\frac{s^2}{t}\,, \tag{40}$$

and the time delay is the same we have computed for the four graviton amplitudes. Therefore, the nonlocal theory (35) is causal as well as the local Einstein-Maxwell-scalar theory.

One could wonder whether causality can still be preserved in a theory where the nonlocality explicitly shows up in the amplitudes, i.e. in cases where the field redefinition theorem cannot be applied. An indication that this is actually possible comes from the theory whose action consists of (2) and the minimally coupled ordinary two-derivatives scalar matter,

$$S = S_{\rm g} + \int d^D x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right). \tag{41}$$

Using the graviton propagator (8), the tree-level gravitational scattering amplitude for 2-scalars in 2-scalars can be easily obtained (we here assume $m=0$):

$$A_s = -8\pi G\frac{ut}{s}e^{-H(s)}\,,\quad A_t = -8\pi G\frac{s(s+t)}{t}e^{-H(t)}\,,$$

$$A_u = -8\pi G\frac{st}{u}e^{-H(u)}\,. \tag{42}$$

In the Regge limit $t \ll s$the leading contribution comes from the amplitude in the t-channel, namely

$$A_t(s,t) \approx -8\pi G\frac{s^2}{t}e^{-H(t)}\,. \tag{43}$$

For $D > 4$ there are no issues related to infrared divergences and we can now compute the phase (25) in $D = 5$,

$$\begin{aligned} \delta(b,s) &= \frac{1}{2s}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{b}} A_t(s,-\vec{q}^2) \\ &= 4\pi G s \int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{b}}\, \frac{e^{-H(-\vec{q}^2)}}{\vec{q}^2} \\ &= \frac{2Gs}{\pi}\int dq \frac{\sin(bq)}{bq}\, e^{-H(-q^2)}\,, \end{aligned} \tag{44}$$

where $q = |\vec{q}|$. In particular, for the form factor

$$e^{-\sigma\Box}\,, \tag{45}$$

which emerges naturally in string field theory [29, 30, 31, 32], one finds the analytic result

$$\delta(b,s)_{\mathrm{SFT}} = Gs\,\frac{\mathrm{Erf}(b/2\ell_\Lambda)}{b}\,, \tag{46}$$

which reduces to the one in Einstein's theory for $b \gg \ell_\Lambda$, namely

$$\delta(b,s)_{\mathrm{SFT}} \quad\to\quad \delta_{\mathrm{EH}}(b,s) = \frac{Gs}{b}. \tag{47}$$

The corresponding time delays are

$$\Delta t_{\mathrm{SFT}} = \frac{16EG}{\pi}\,\pi\frac{\mathrm{Erf}(b/2\ell_\Lambda)}{b}\,, \tag{48}$$

$$\Delta t_{\mathrm{EH}} = \frac{16EG}{\pi}\,\frac{\pi}{b}\,. \tag{49}$$

In Fig.1 we plot Δt_{T} for the form factor (5), which has been obtained numerically, together with Δt_{SFT} and Δt_{EH}. Very similar results can be obtained for different values of α and γ

From the analytical results as well as from the plots, it is clear that the Shapiro's time delay never becomes negative and the causality condition is satisfied up to and beyond the non locality scale ℓ_Λ.

5. Conclusions

Weakly nonlocal theories are an interesting arena where such crucial ideas about quantum gravity as ultraviolet finiteness, perturbative unitarity and causality can be tested in a very straightforward way thanks to the powerful formalism of quantum field theory. In particular we have given evidence that in a lot of cases we can extend Einstein-Hilbert gravity without violating

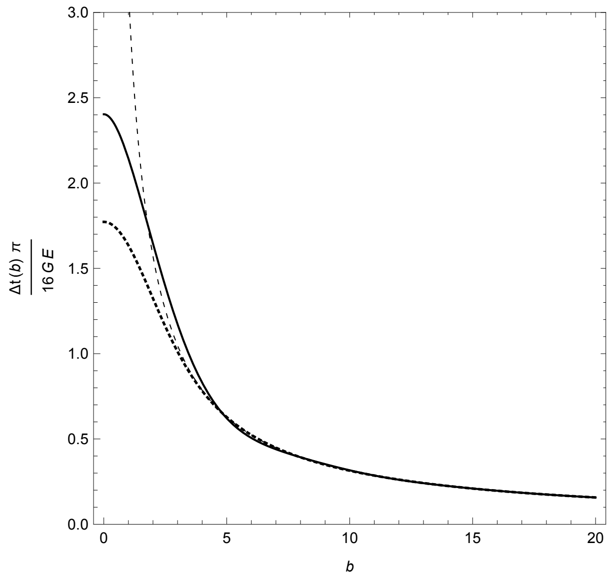

Figure 1: From top to bottom the lines represent respectively the following Shapiro's delays: Δt_{EH}, Δt_{T} (for $\gamma = 3$), and Δt_{SFT}. We also assumed $\ell_\Lambda = 1$.

the notion of causality related to Shapiro's time delay and discussed in [1]. Contrary to what happens in weakly coupled string theory, causality is not achieved by the introduction of an infinite tower of massive higher spin fields, but by avoiding the higher-derivative terms which could cause a Shapiro time advance. This has been proven to be possible in several cases. In particular a field redefinition theorem allows to construct a wide class of nonlocal theory for matter coupled to gravity compatible with causality. As a particular applications of the theorem, we have discussed the Einstein-Maxwell-Scalar nonlocal field theory, which can be proven to be causal, unitary, and finite in the ultraviolet. Other examples discussed in [24] are the $N = 1$ nonlocal supergravity [28] and Lee-Wick gravity [33, 34, 35, 36, 37]. In general, causality represents a valuable guide principle to understand what kind of higher derivative terms can show up in a consistent quantum gravity theory.

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