

Holography for heavy ions collisions

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Outlook



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- Holography for QCD (**AdS/QCD**)
- Holography for HIC (**Heavy-Ions Collisions**)

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I.A, “Holographic approach to quark–gluon plasma in heavy ion collisions”,
Phys. Usp., 57:6 (2014), 527–555

I.A, “Holography for Heavy Ions Collisions at LHC and NICA”, 1612.08928

Few remarks on holography (AdS/CFT)



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Holography translates the physics of quantum many body systems into a dual classical gravitational problem in a space-time with an extra dimension.

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Perturbation theory doesn't reproduce experimental value

Arnold, Moore, and Yaffe (2000)

$$\frac{\eta}{s} \sim \frac{1}{g_{\text{YM}}^4 \log g_{\text{YM}}^{-1}}$$

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Policastro, Son, Starinets, PRL'01

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III) Multiplicity

Landau theory predicts

$$\mathcal{M}_L \sim s^{0.25}$$

IA, Golubtsova, '15

$$\mathcal{M}_{HQCD} \sim \mathcal{M}_{LHC} \sim s^{0.155}$$

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 - **Motivations for the model**
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I.A., K.Rannu, JHEP 1805 (2018) 206

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I.A., K.Rannu, JHEP 1805 (2018) 206

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- What is special for NICA

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- 4) The action has to be found by «trial and error method»
- 5) What is a «best» 5-dim background?

5-dim Anisotropic Background

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Einstein-dilaton-two-Maxwell

I.A., K. Rannu, JHEP' 18

$$S = \int \frac{d^5x}{16\pi G_5} \sqrt{-\det(g_{\mu\nu})} \left[R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]$$

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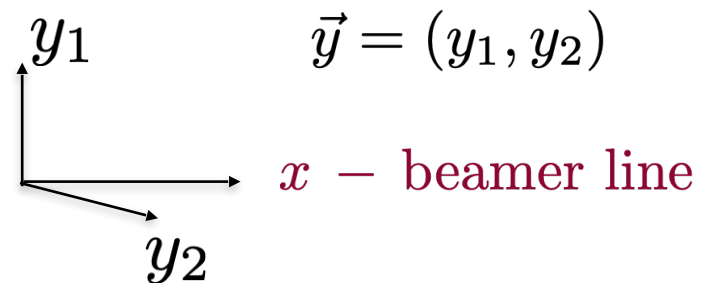
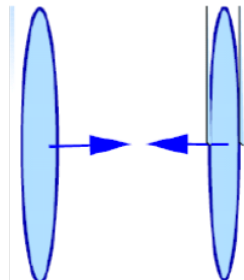
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Schematic picture of central HIC



Anisotropic thermalization



QGP is created after very short time after the collision $\tau_{therm} \sim 0.1 fm/c$ and it is anisotropic for a short time $0 < \tau_{therm} < \tau < \tau_{iso}$

The time of locally isotropization is about $\tau_{iso} \sim 2 fm/c$

Heavy Ions Collisions in Holographic Approach: shock waves collisions in 5-dimension



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$$ds^2 = \frac{1}{z^2} \left(-dt^2 + dx^2 + z^{2-2/\nu} (dy_1^2 + dy_2^2) + dz^2 \right)$$

IA, Golubtsova, JHEP'15

Shock domain walls/planar shocks collision:

ENTROPY

$$\mathcal{M} = \frac{\nu}{2G_5} (8\pi G_5)^{1/(1+\nu)} s^{\frac{1}{2+\nu}}$$

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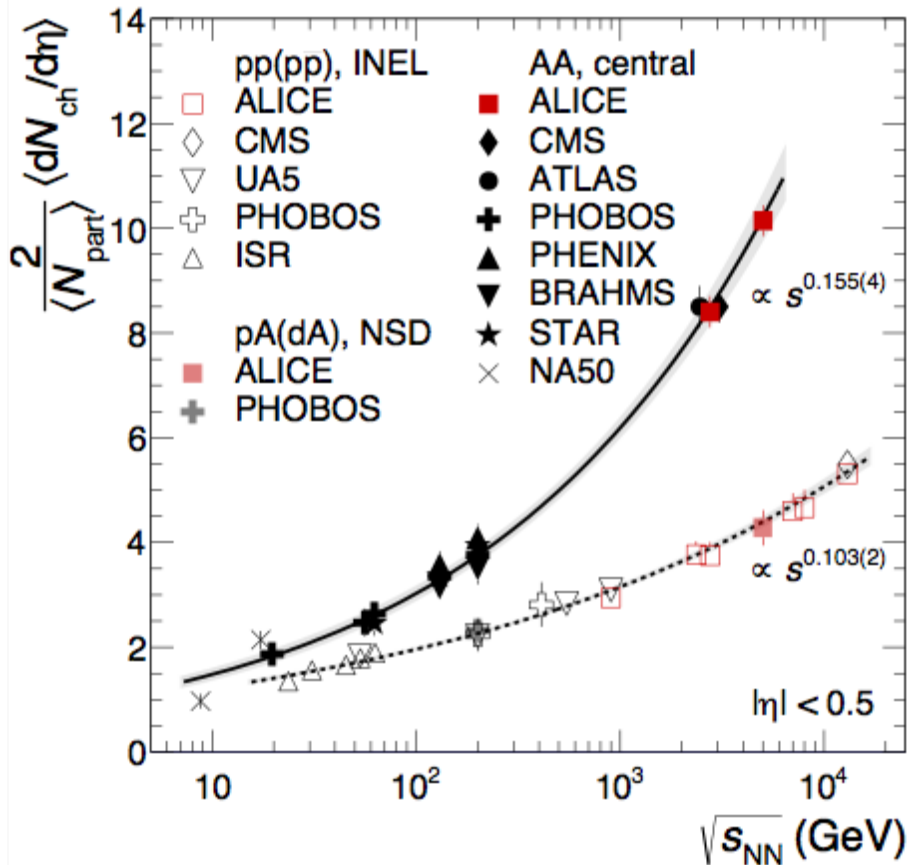
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$$\nu = 4.45$$

Longitudinal-transversal anisotropy.

Motivation: Multiplicity

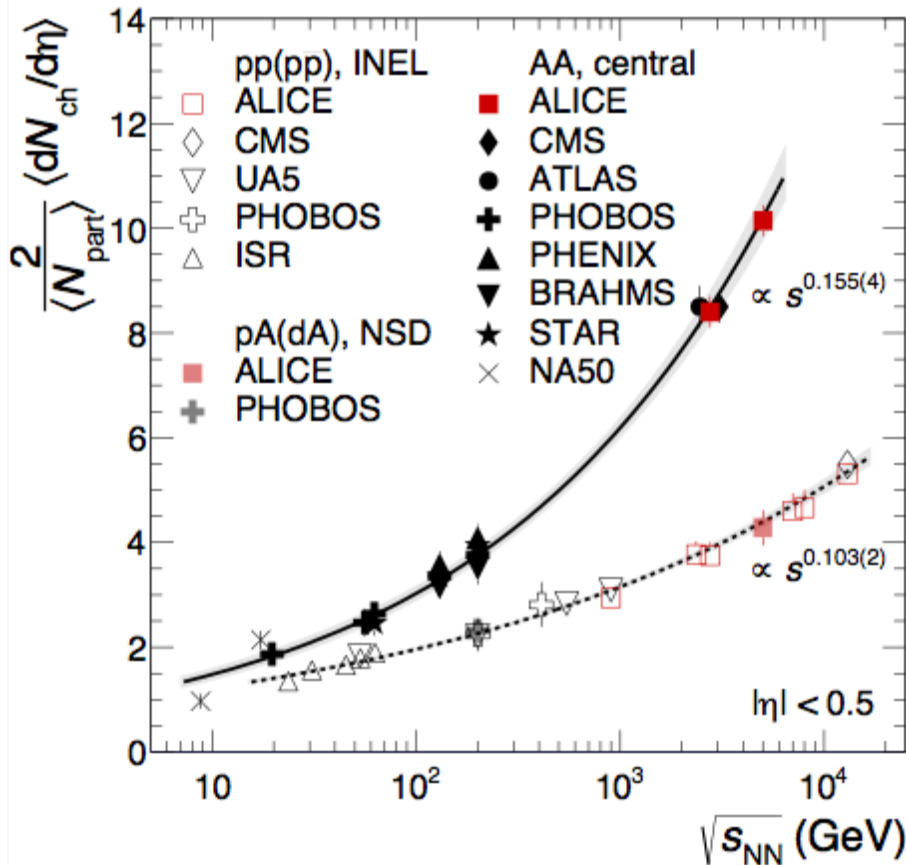


Plot from **PRL'16**
(ALICE).

$$\mathcal{M}_{LHC} \sim s^{0.155}$$

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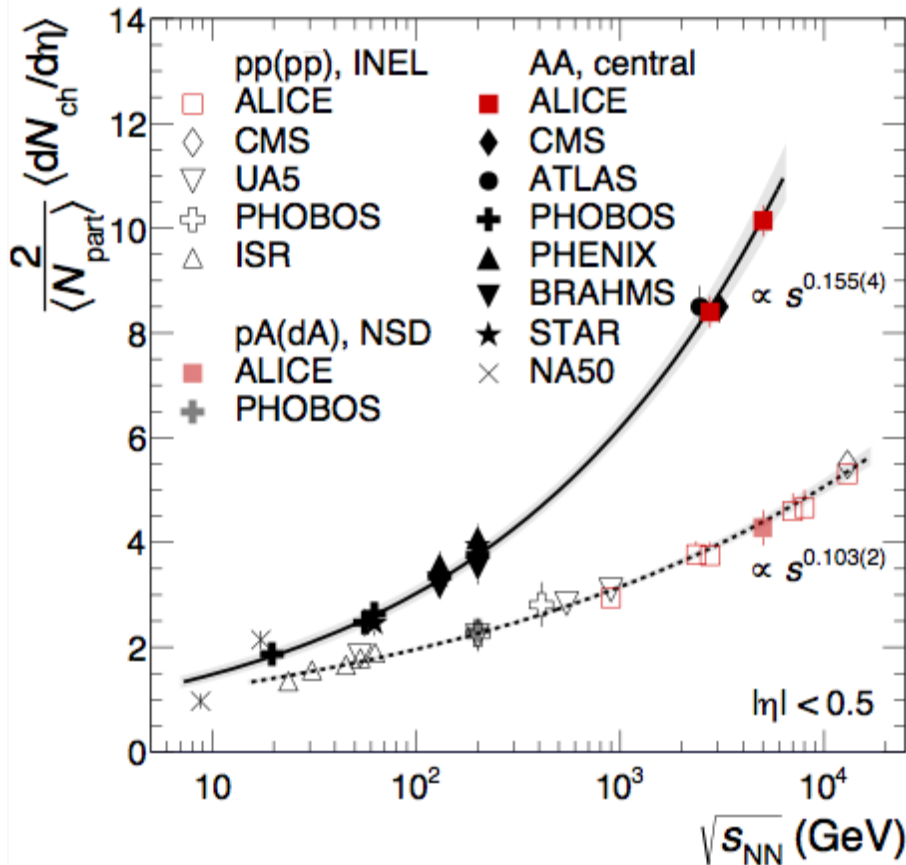
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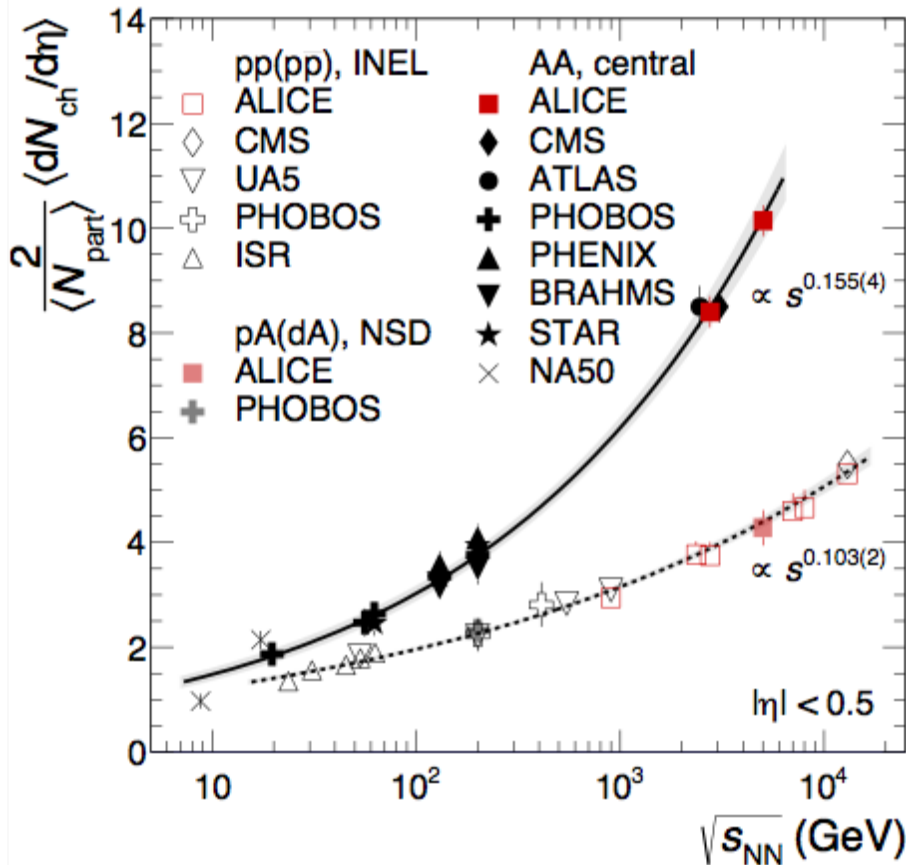
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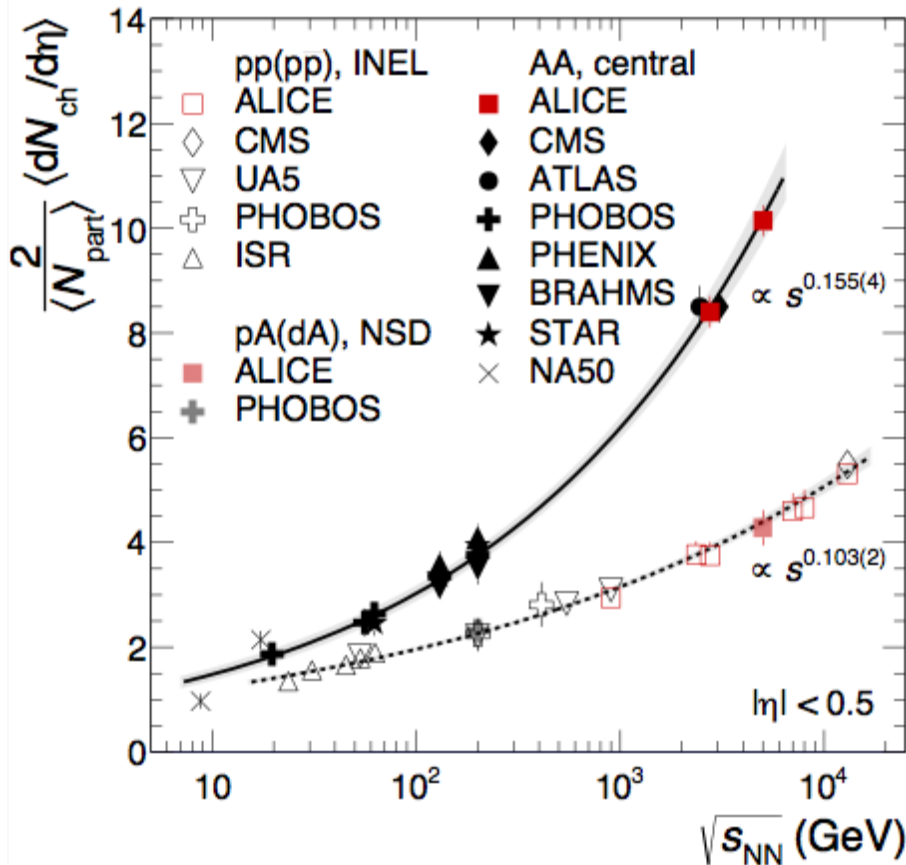
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Quark-Gluon Plasma (QGP): a new state of matter

QGP is a state of matter formed from deconfined quarks, antiquarks, and gluons at high temperature

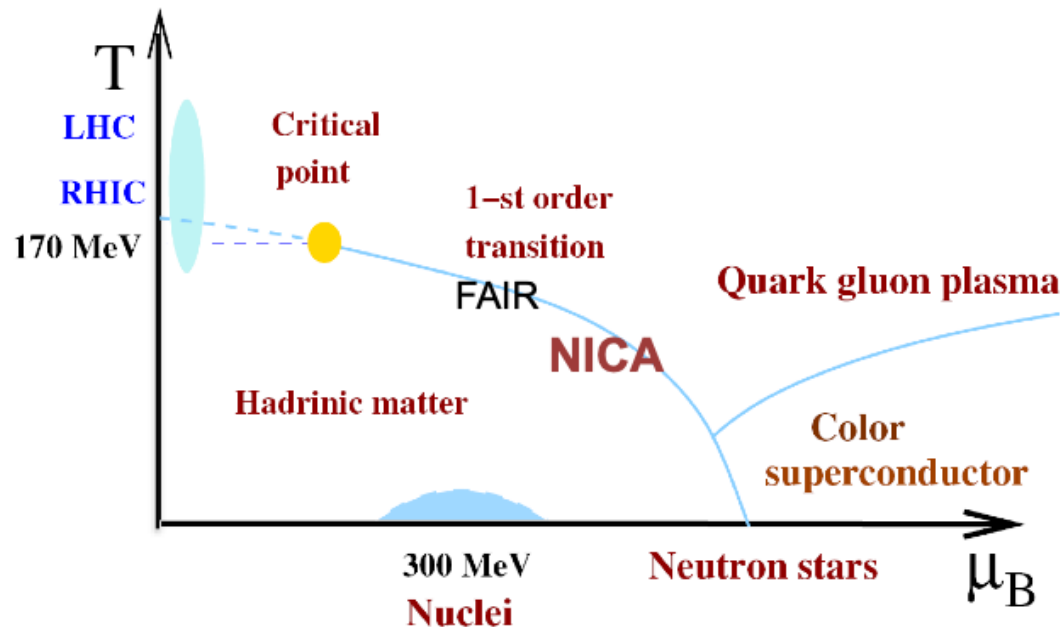
QCD: asymptotic freedom, quark confinement

T increases, or
density increases

nuclear
matter



Deconfined
phase



5-dim Anisotropic Background

Einstein-dilaton-two-Maxwell

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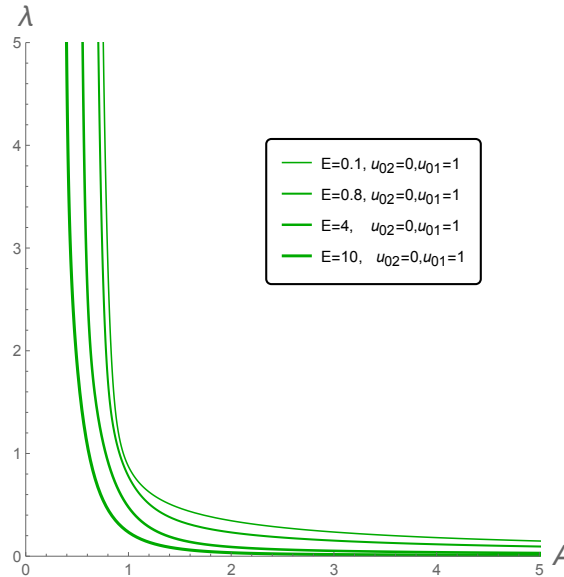
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“Reconstruction of potential” for zero chemical potential

**White, 0701157; Pirner, Galow, 0903.2701;
He, Wu, Yang, Yuan, 1301.0385,
M.-W. Li, Y. Yang arXiv:1703.09184**

Holographic RenormGroup Flow, T=0

$$\beta(\lambda) = \frac{d\lambda}{d \log E} = \lambda \frac{d\phi}{d \log \mathcal{B}}, \quad \mathcal{B} = \frac{\sqrt{b(z)}}{z}.$$



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$$V_0 = -0.6, \quad K_1 = 0.8, \quad K_2(4.5) = 2.1$$

$$C_1 = 23, \quad C_2 = 0.06$$

I.A., Rannu, 1802.05652

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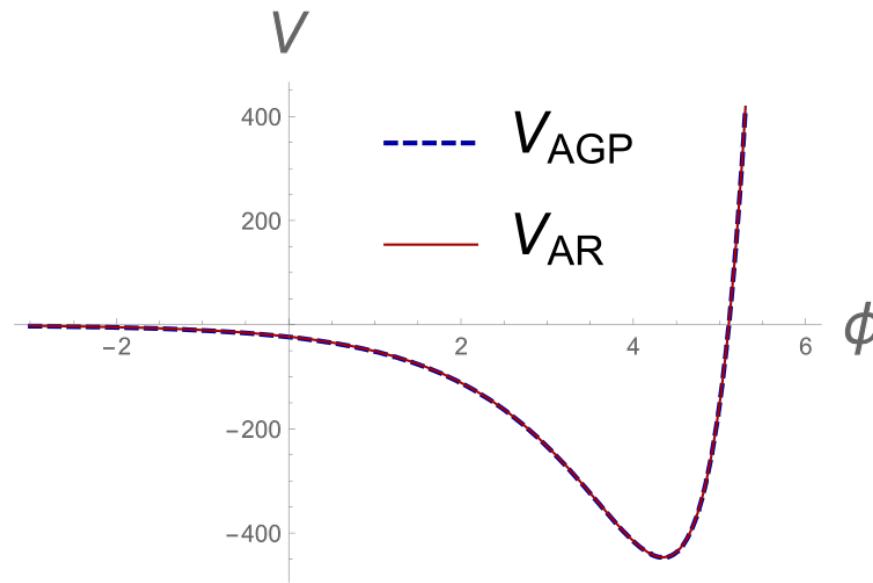
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I.A., Golubtsova, Policastro, arXiv: 1803.06764

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**Smearing confinement/deconfinement
phase transition (crossover transition)
in holography for the anisotropic model**

Temporal Wilson loops



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Energy between quarks located along x-direction

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Holography for a probe

$$S_{xt} = \frac{1}{2\pi\alpha'} \int d\sigma^1 d\sigma^2 \sqrt{-\det(h_{\alpha\beta})}$$

The recipe by Maldacena ('98), Rey et al ('98),
Sonnenschein et al ('98)

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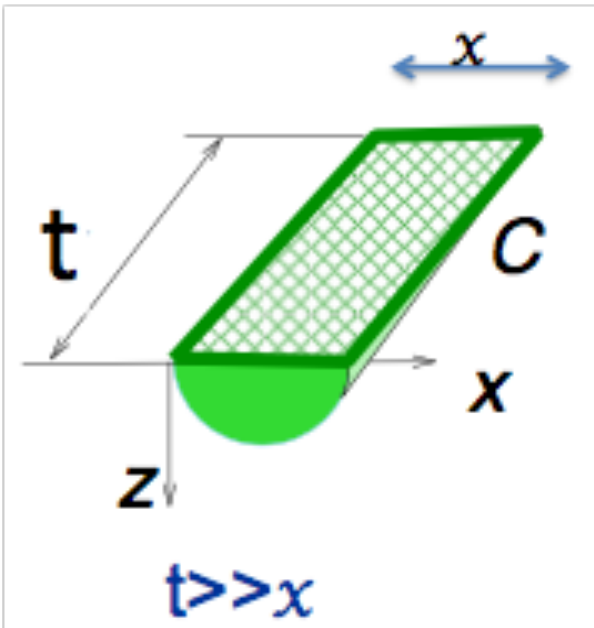
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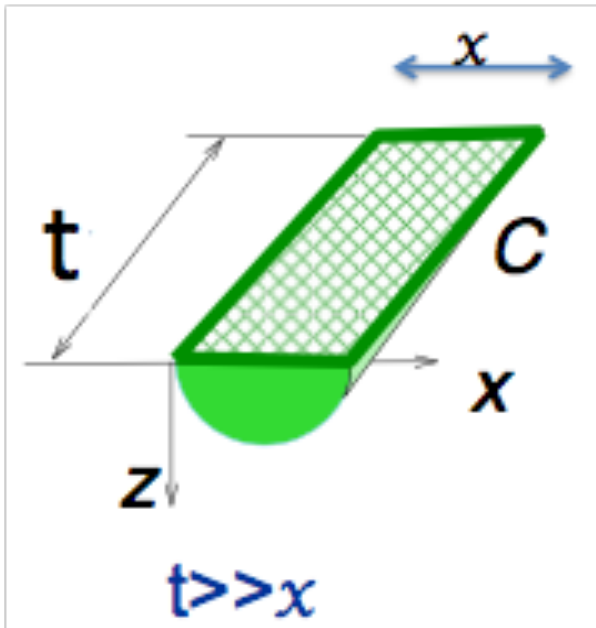
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The recipe by Maldacena ('98), Rey et al ('98),
Sonnenschein et al ('98)



Temporal Wilson loops

Energy between quarks located along x-direction



$$W(T, X) = \langle \text{Tr}_F e^{i \oint_{\Gamma \times X} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}$$

Holography for a probe

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Contour approach in YM in 80'

Temporal Wilson loops

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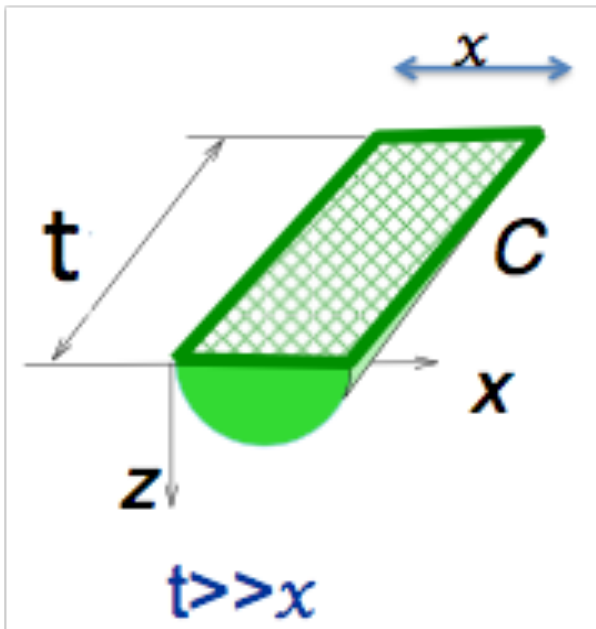
$$W(T, X) = \langle \text{Tr}_F e^{i \oint_{\Gamma_{TX}} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}$$

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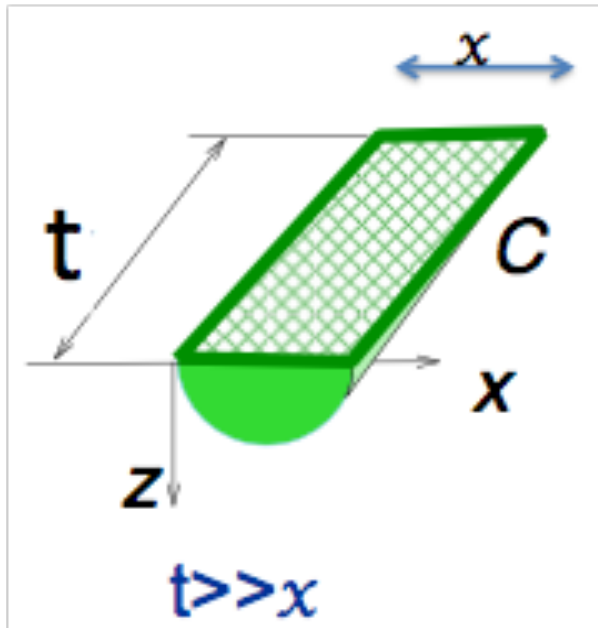
2-dim Born-Infeld dynamical system



Contour approach in YM in 80'

Temporal Wilson loops

Energy between quarks located along x-direction



Contour approach in YM in 80'

$$W(T, X) = \langle \text{Tr}_F e^{i \oint_{\Gamma_{t,x}} dx_\mu A_\mu} \rangle \sim e^{-V(X)T}$$

Holography for a probe

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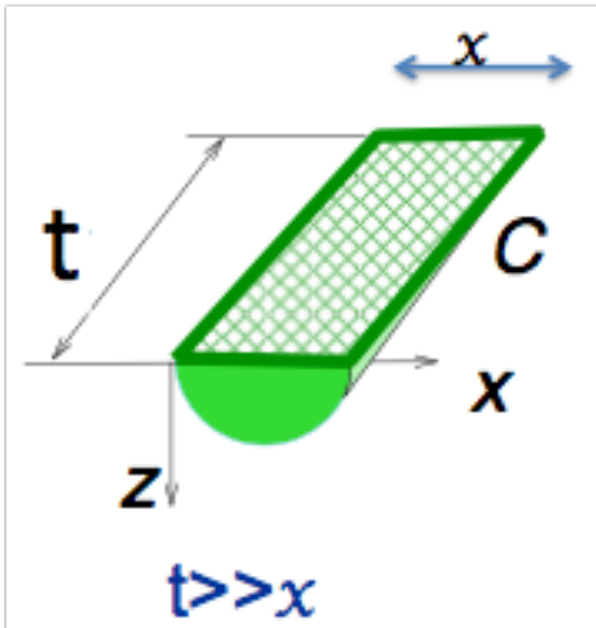
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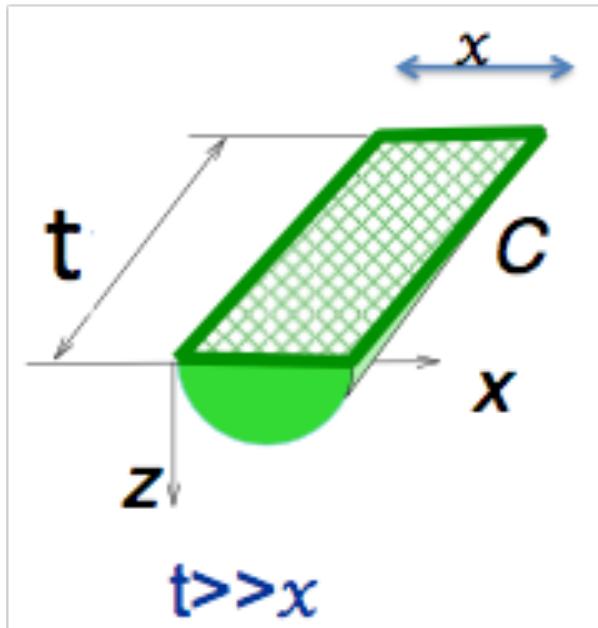
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Temporal Wilson loops

Energy between quarks located along x-direction



Contour approach in YM in 80'

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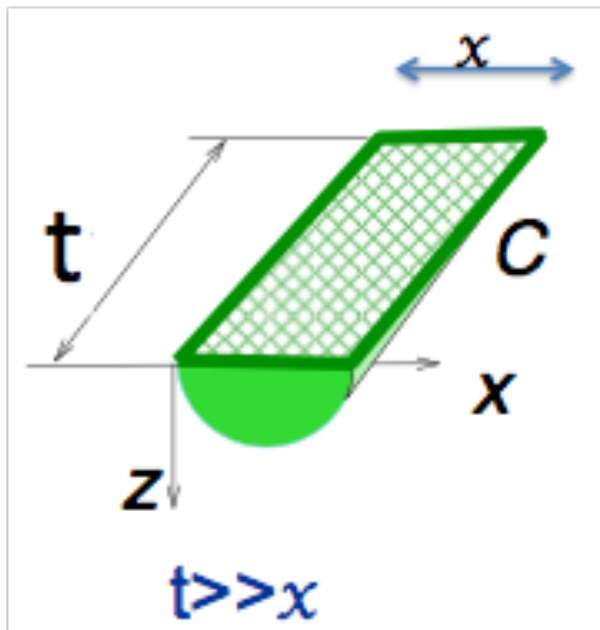
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Effective actions
depend on orientation

Temporal Wilson loops

Energy between quarks located along x-direction



Contour approach in YM in 80'

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Effective actions
depend on orientation

$$\mathcal{V}_x = \frac{e^{P(z) + \sqrt{\frac{2}{3}}\phi(z)}}{z^2} \sqrt{g(z)},$$

$$\mathcal{V}_y = \frac{e^{P(z) + \sqrt{\frac{2}{3}}\phi(z)}}{z^{1/\nu+1}} \sqrt{g(z)}.$$

Dynamical domain wall

Dynamical domain wall

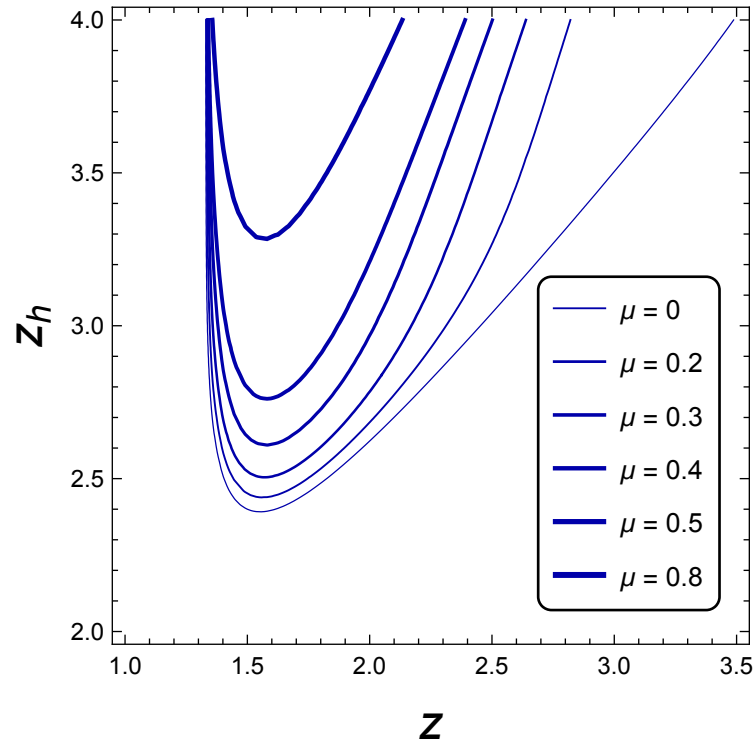
$$\mathcal{V}'_x(z_{DW_x}) = 0$$

$$\mathcal{V}'_y(z_{DW_y}) = 0$$

Dynamical domain wall

$$\mathcal{V}'_x(z_{DW_x}) = 0$$

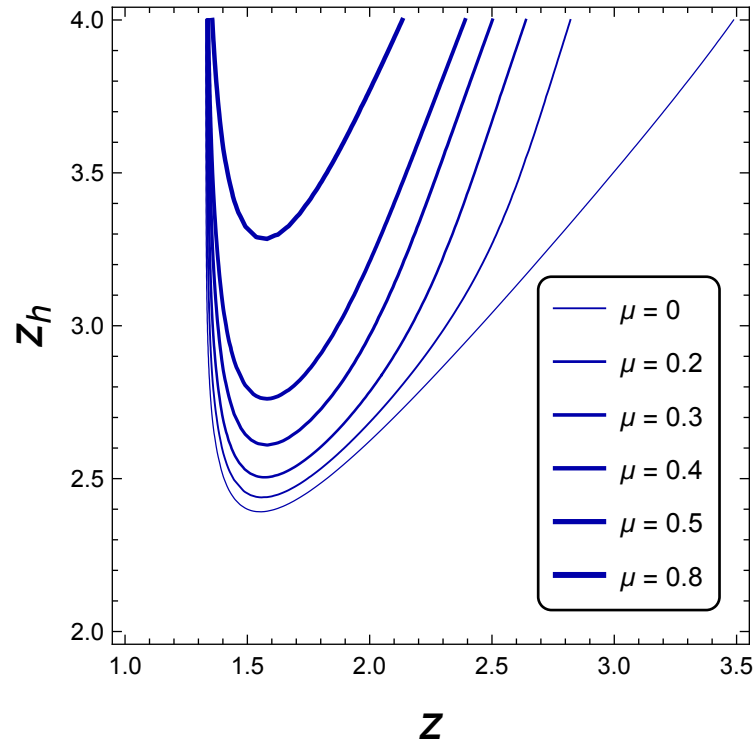
$$\mathcal{V}'_y(z_{DW_y}) = 0$$



Dynamical domain wall

$$\mathcal{V}'_x(z_{DW_x}) = 0$$

$$\mathcal{V}'_y(z_{DW_y}) = 0$$



$$V_{Cornell}(x) \equiv V_{Q\bar{Q}}(x) = -\frac{\kappa}{x} + \sigma_{str}x + V_0$$

Holographic anisotropic QCD phase diagram



Holographic anisotropic QCD phase diagram



$W_x T$

Holographic anisotropic QCD phase diagram



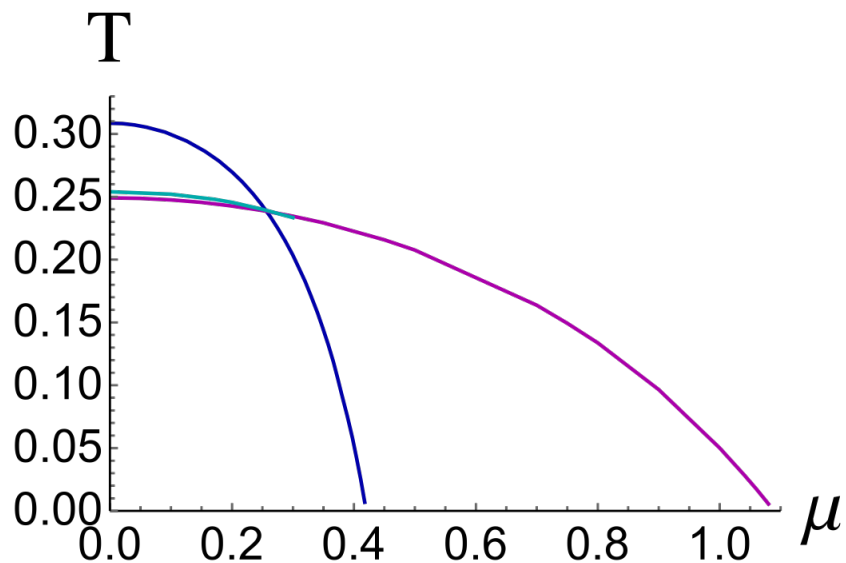
$W_x T$

$W_y T$

Holographic anisotropic QCD phase diagram

$W_x T$

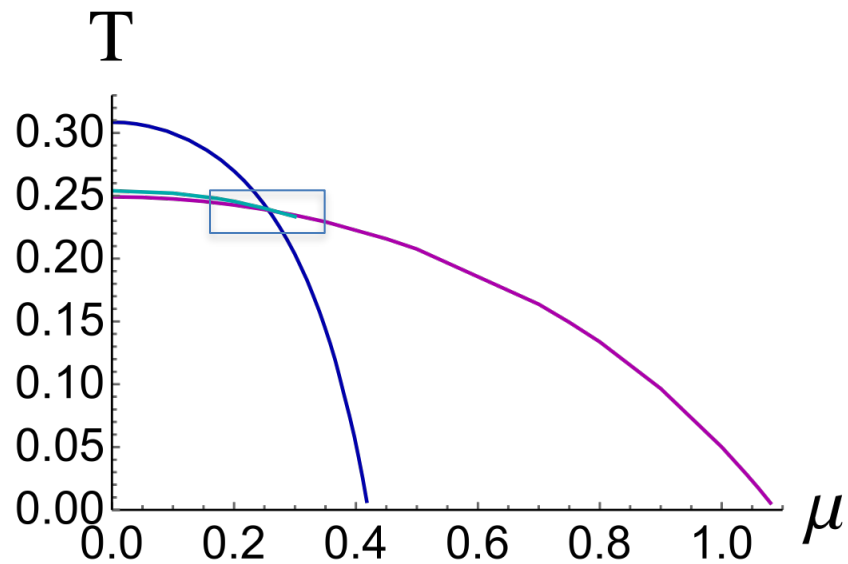
$W_y T$



Holographic anisotropic QCD phase diagram

$W_x T$

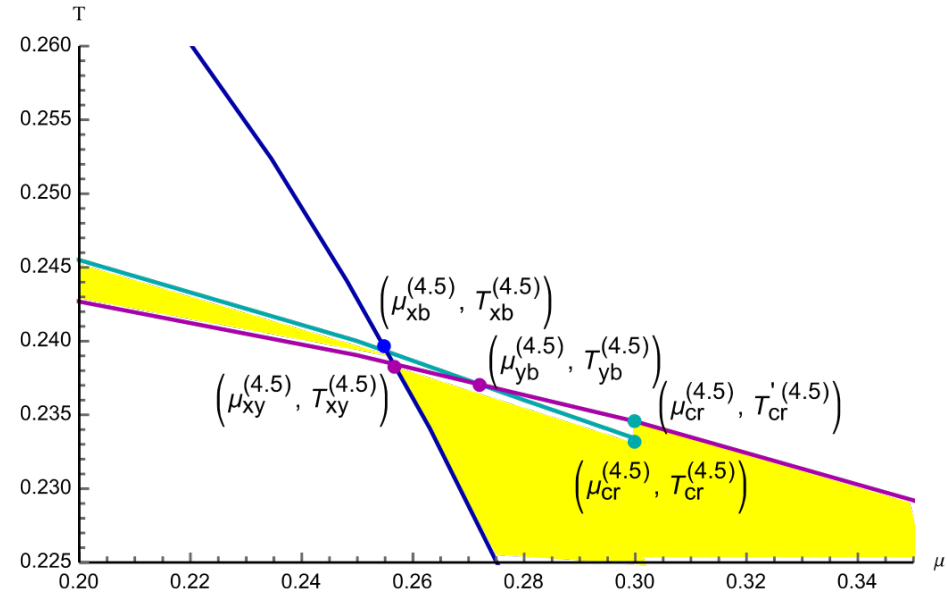
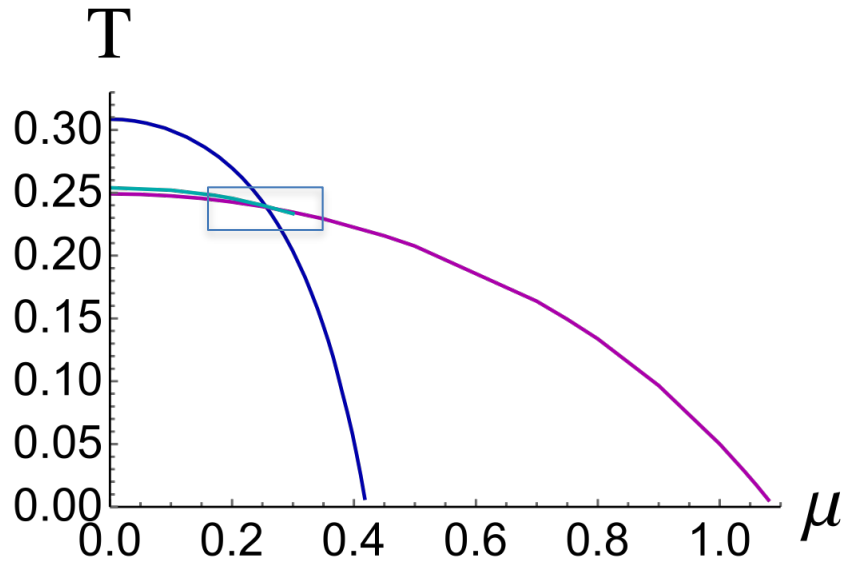
$W_y T$



Holographic anisotropic QCD phase diagram

$W_x T$

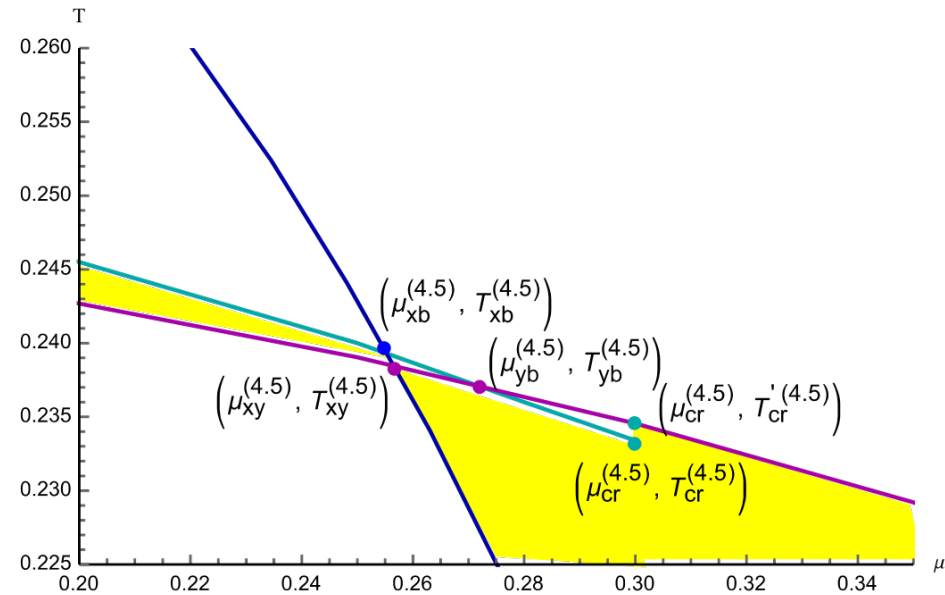
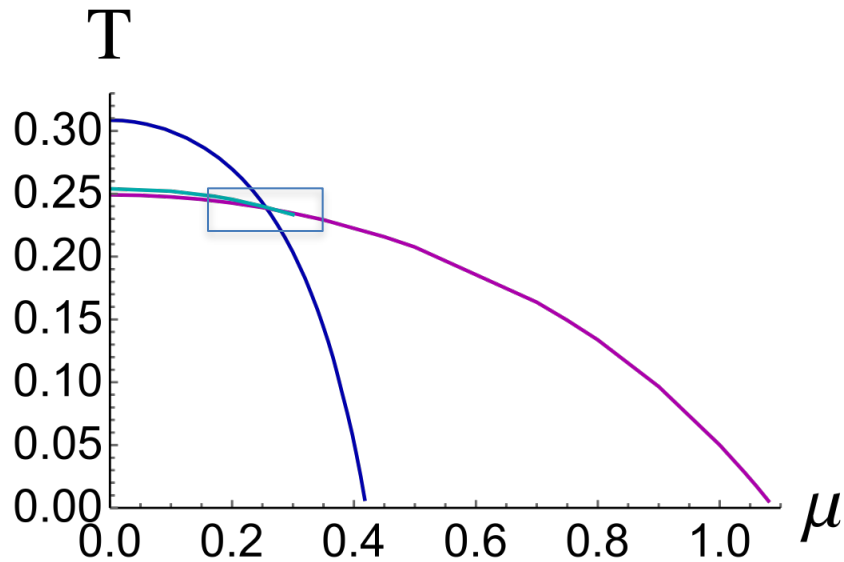
$W_y T$



Holographic anisotropic QCD phase diagram

$W_x T$

$W_y T$

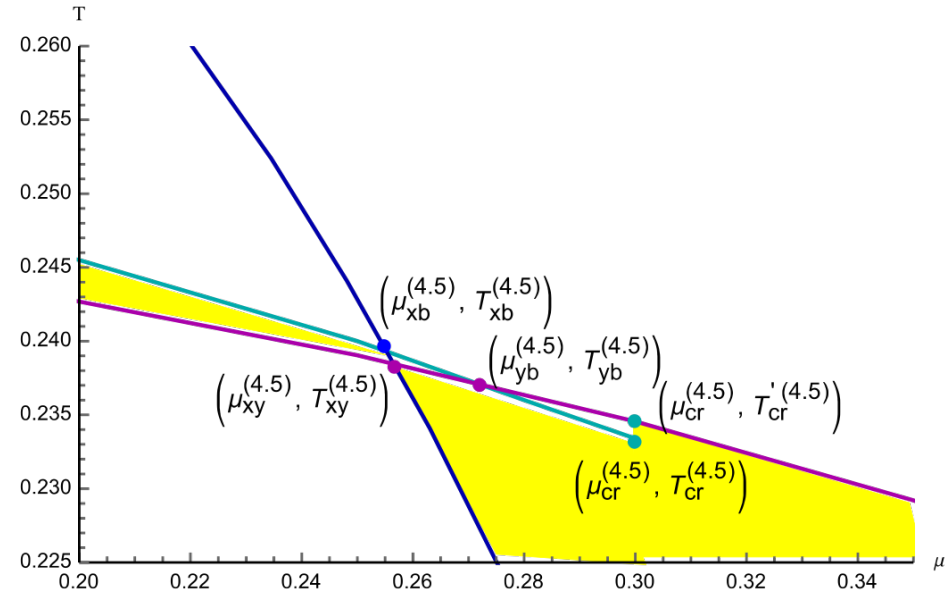
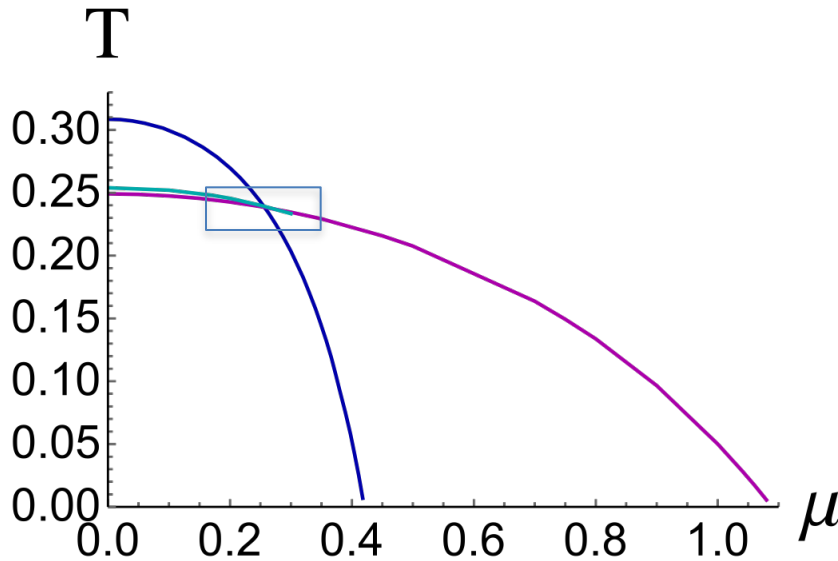


Smearred confinement/deconfinement phase transition

Holographic anisotropic QCD phase diagram

$W_x T$

$W_y T$



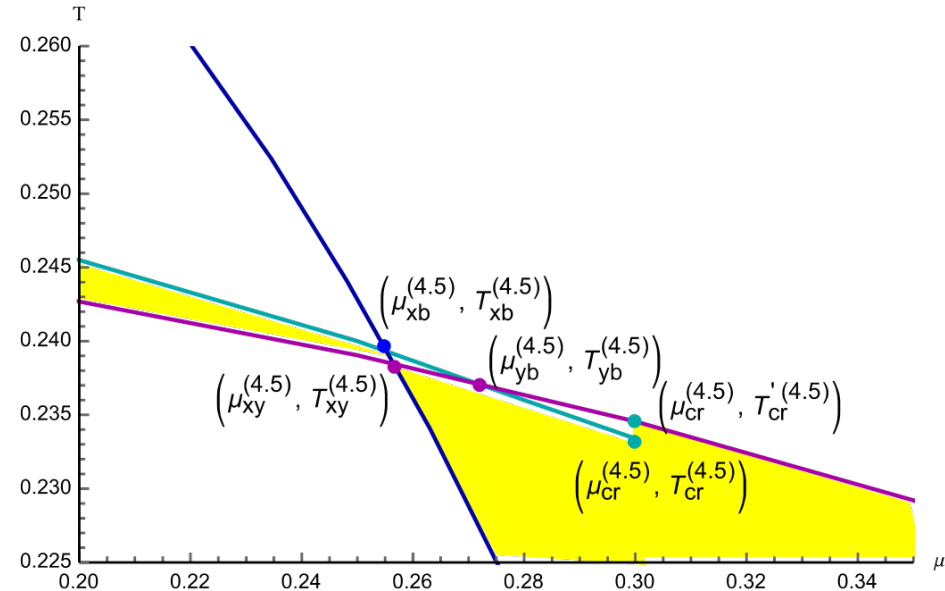
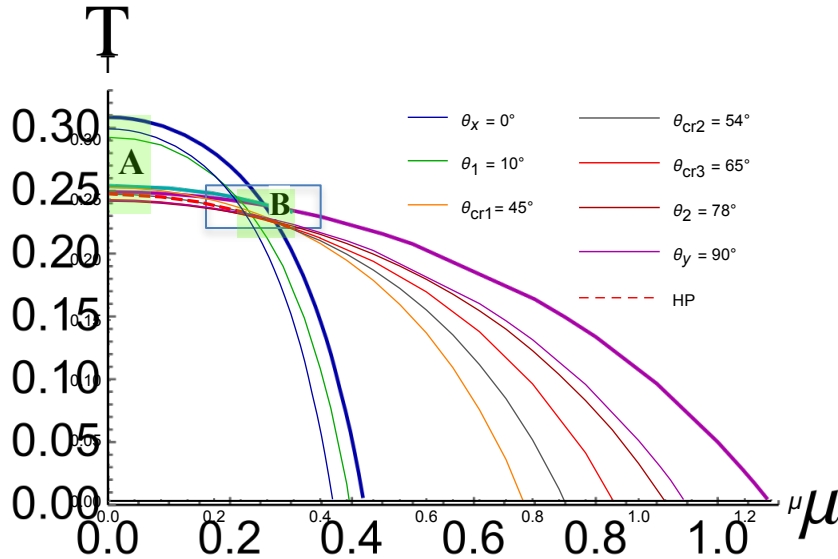
Smearred confinement/deconfinement phase transition

Arbitrary angle: IA, K.Rannu, P.Slepov, PLB'19

Holographic anisotropic QCD phase diagram

$W_x T$

$W_y T$



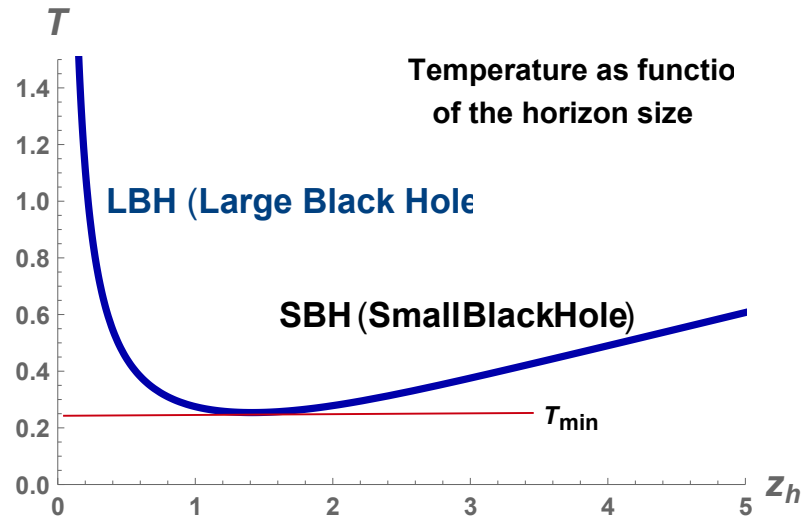
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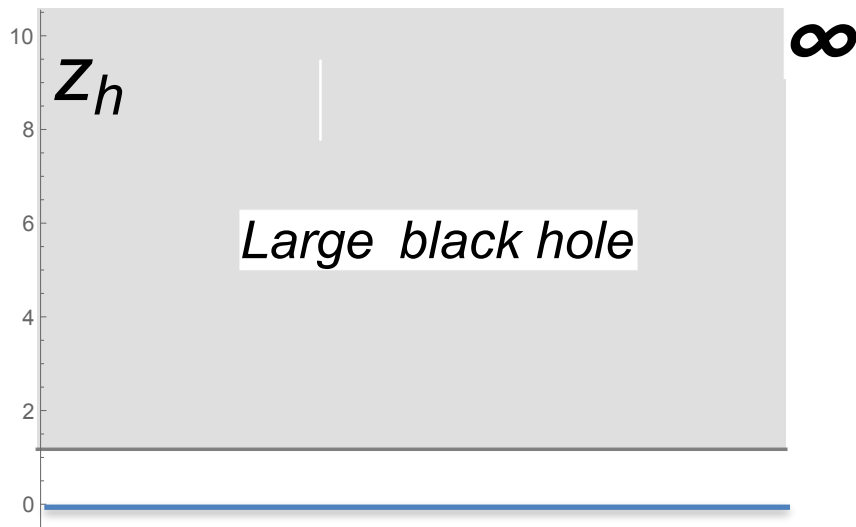
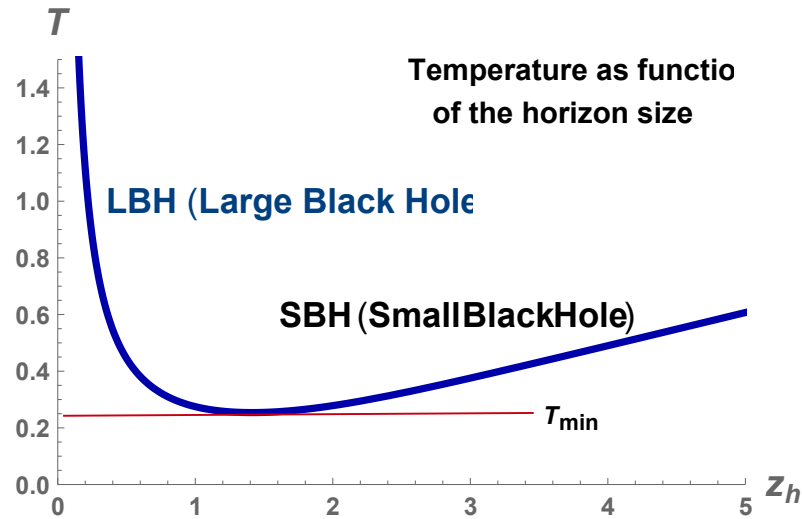
Thermodynamics of the background

Temperature as function of horizon

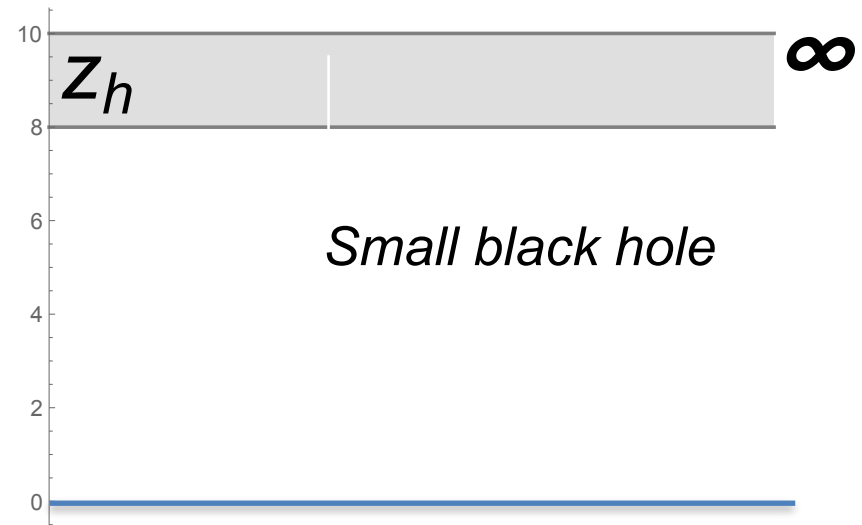
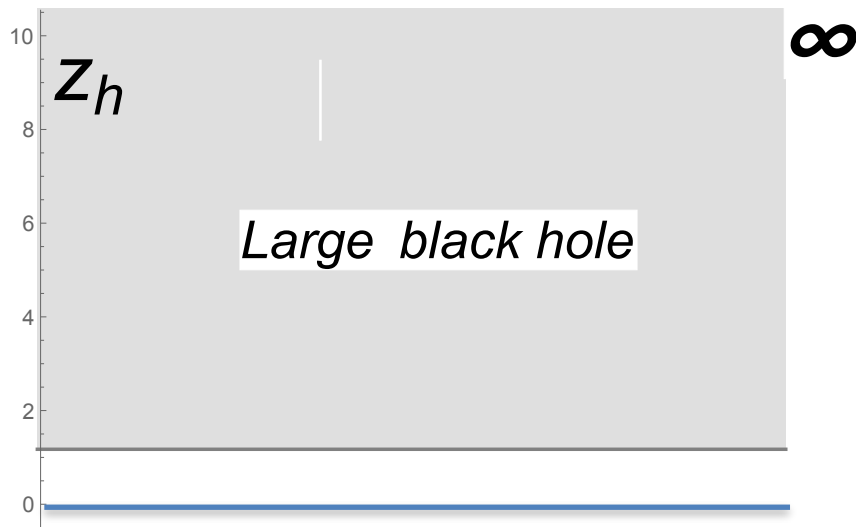
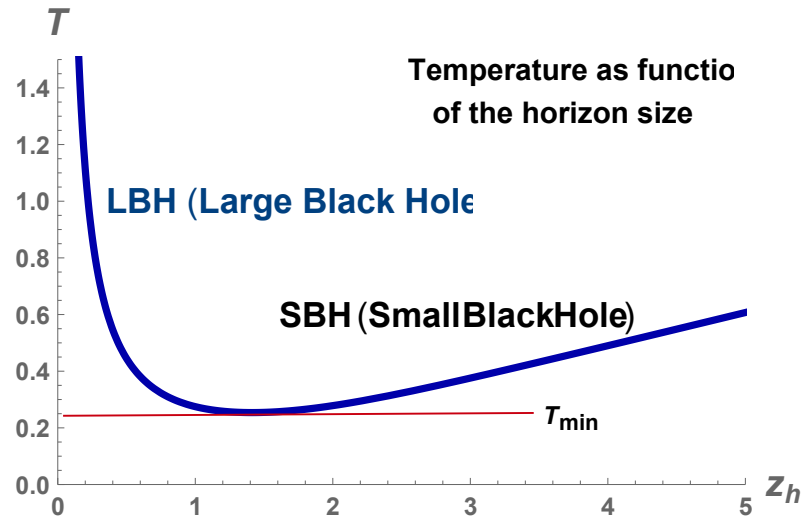
Temperature as function of horizon



Temperature as function of horizon

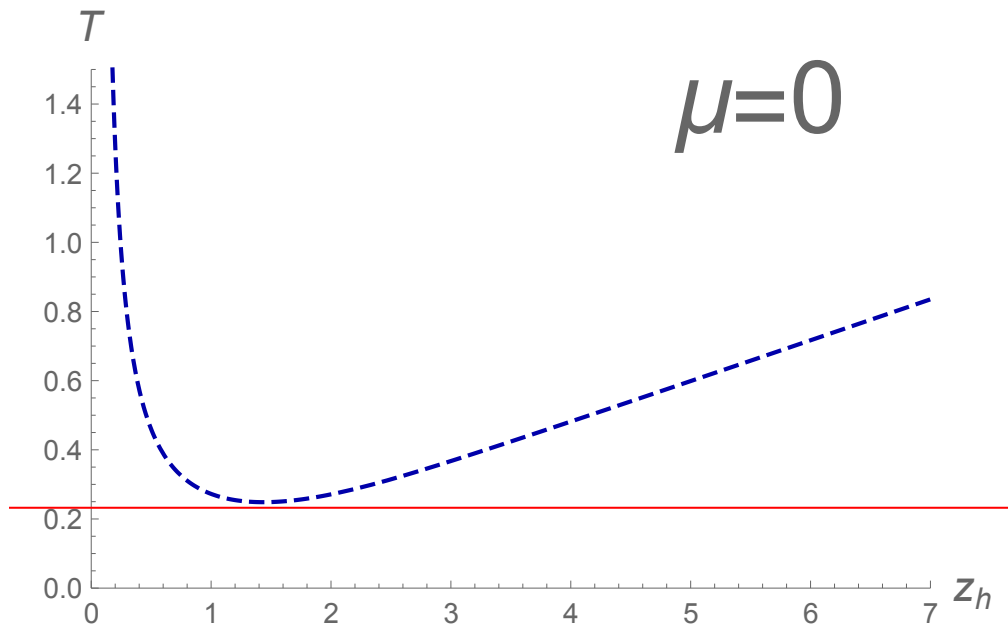


Temperature as function of horizon



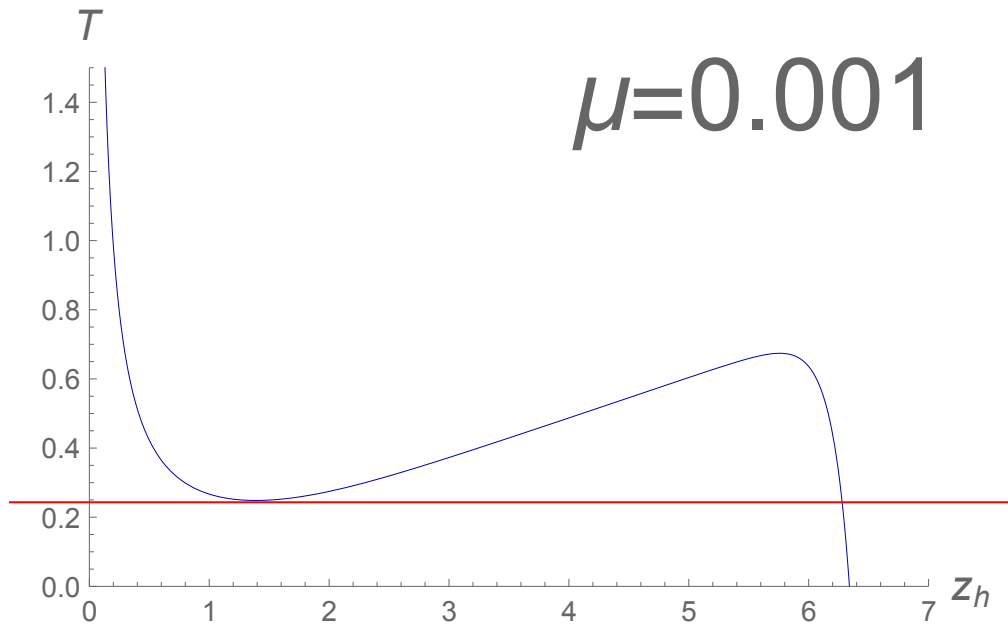
Disappearance of local max and min

Disappearance of local max and min

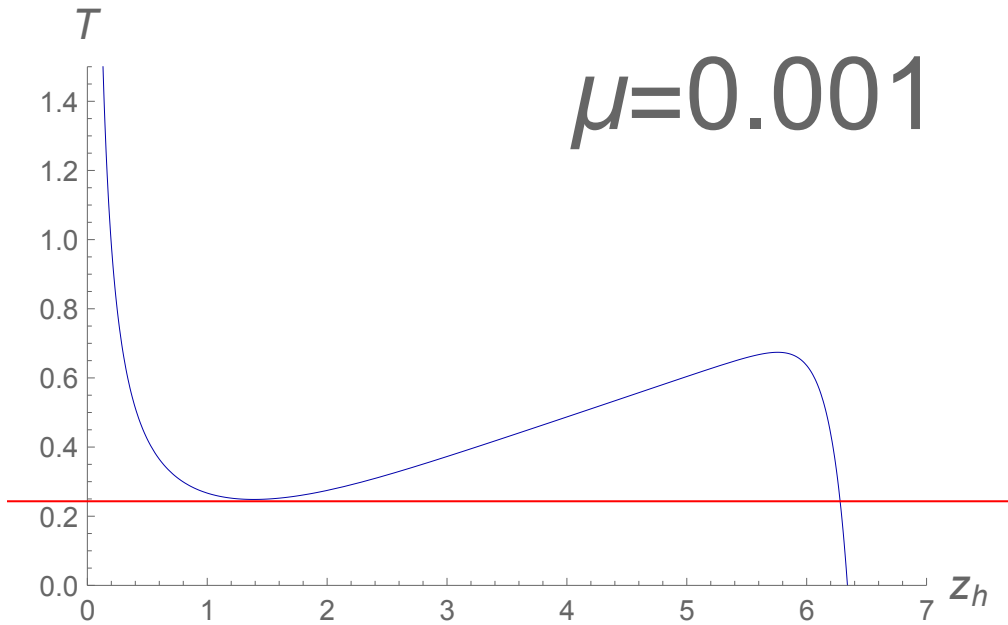


Disappearance of local max and min

Disappearance of local max and min



Disappearance of local max and min

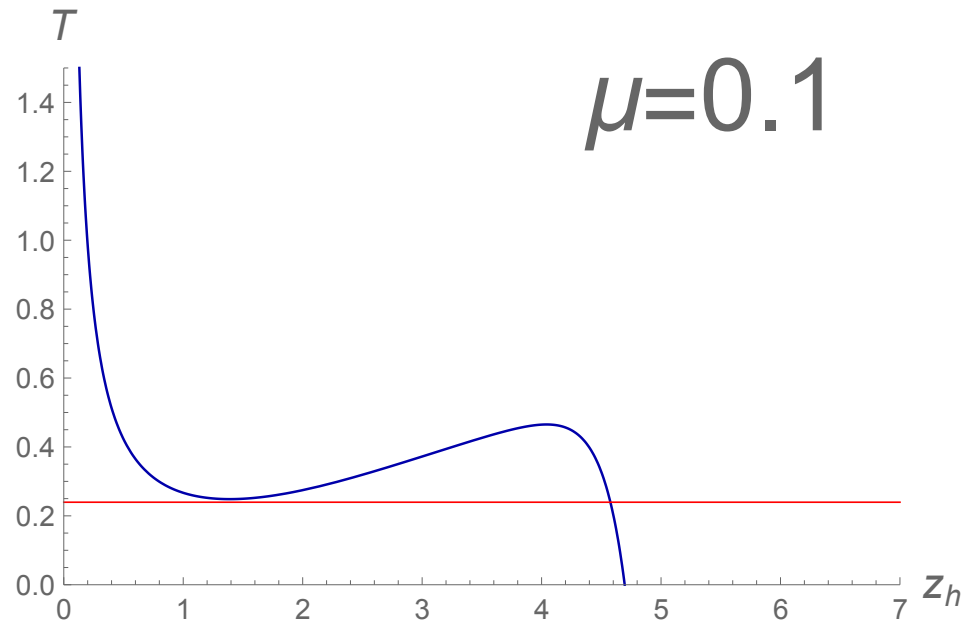


**The appearance of
second horizon**

Disappearance of local max and min

**The appearance of
second horizon**

Disappearance of local max and min

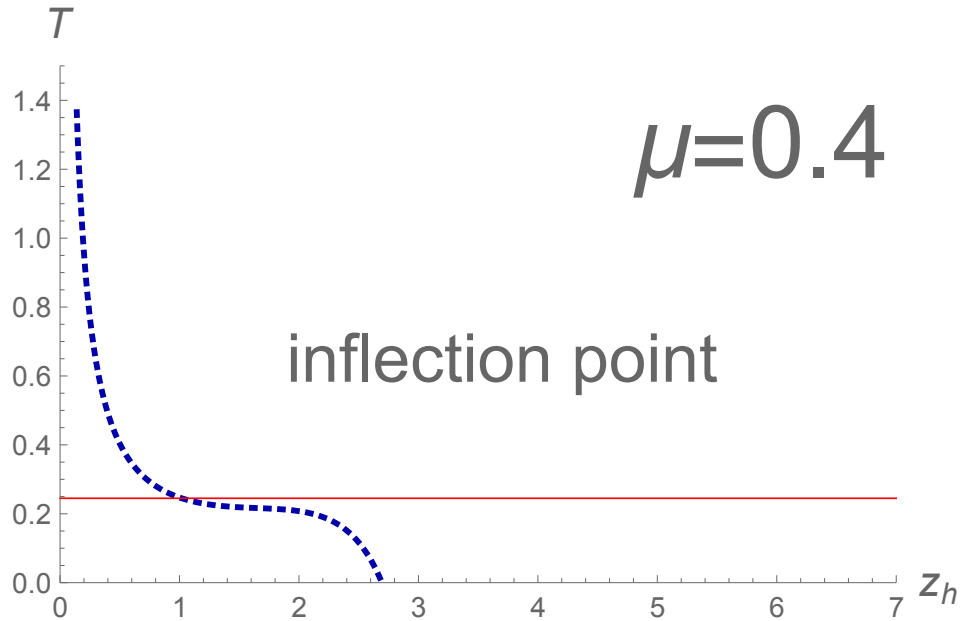


**The appearance of
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Disappearance of local max and min

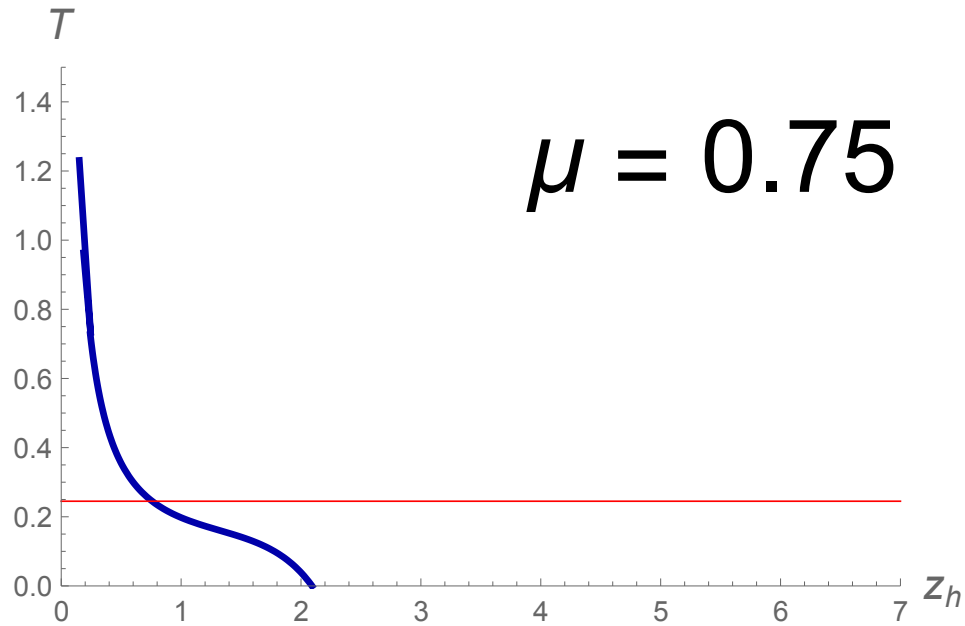
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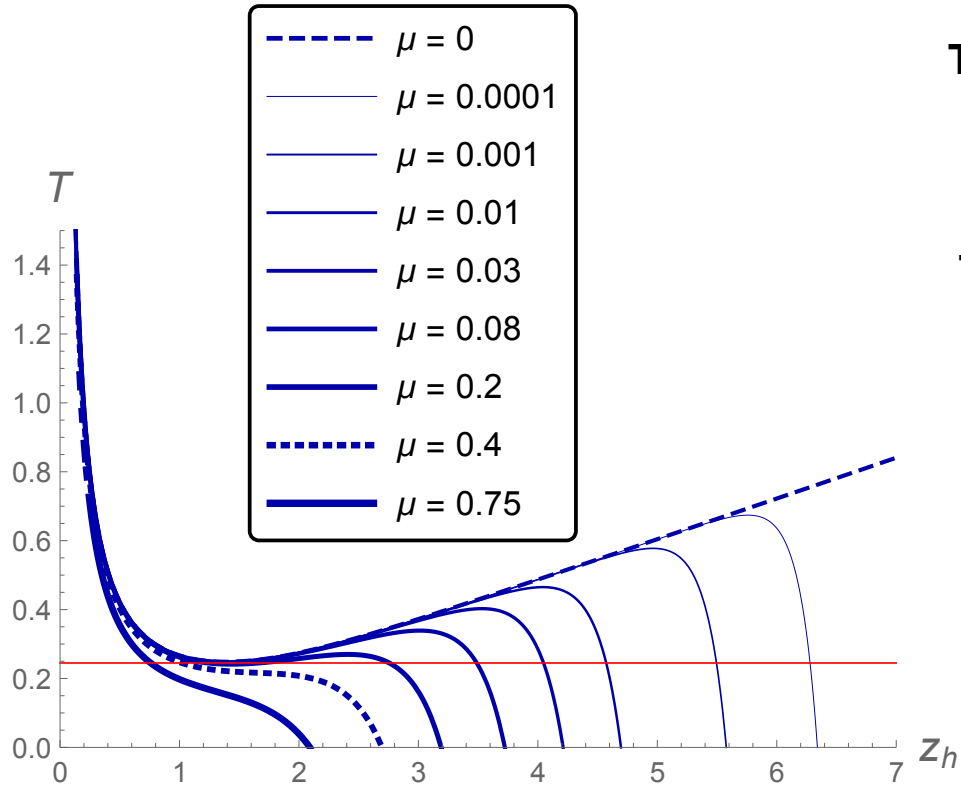
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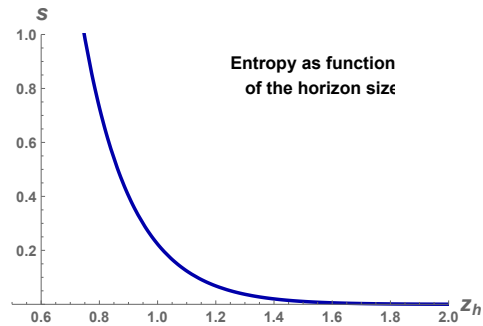
**The disappearance of
local max and min**

Entropy as function of the temperature

$$s(z_h, c, \nu) = \frac{e^{\frac{3}{4}cz_h^2}}{4} z_h^{-\frac{(\nu+2)}{\nu}}$$

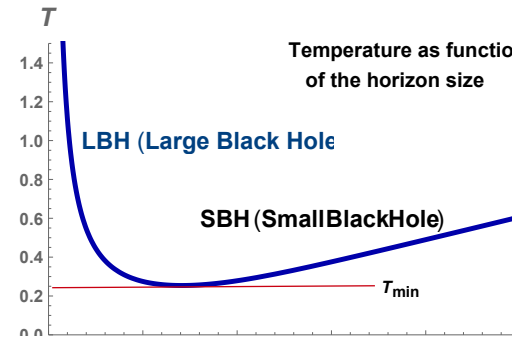
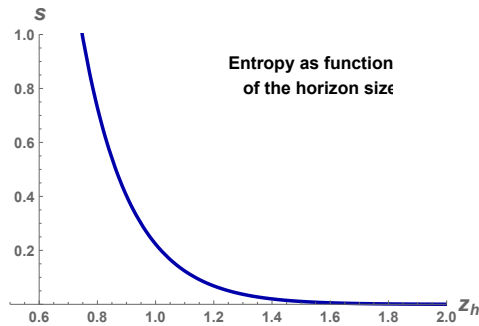
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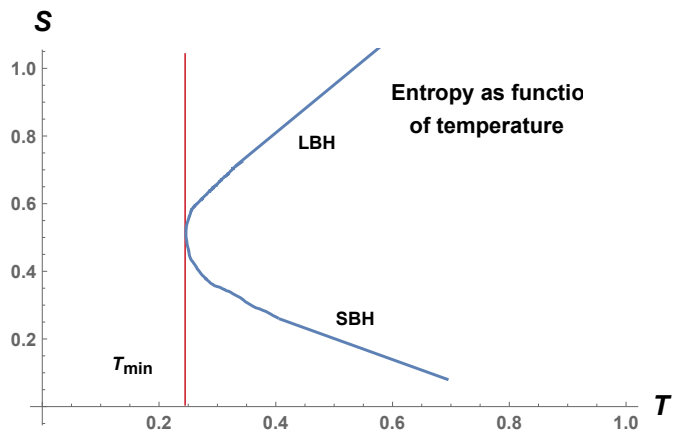
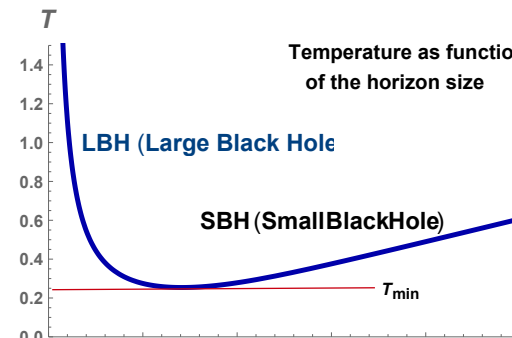
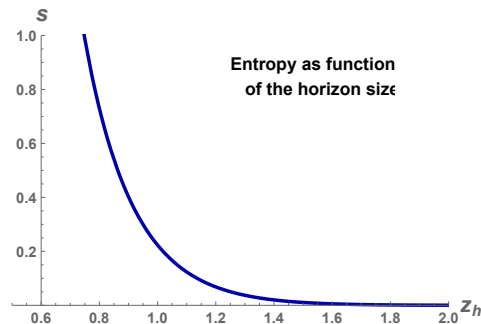
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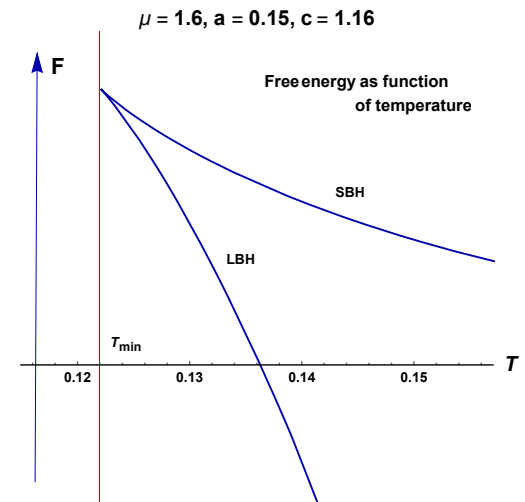
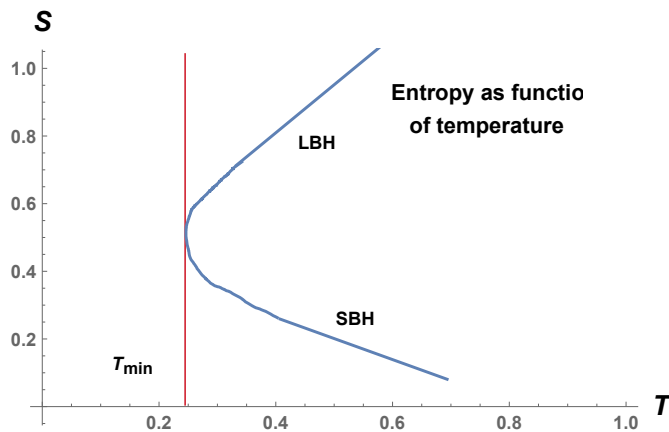
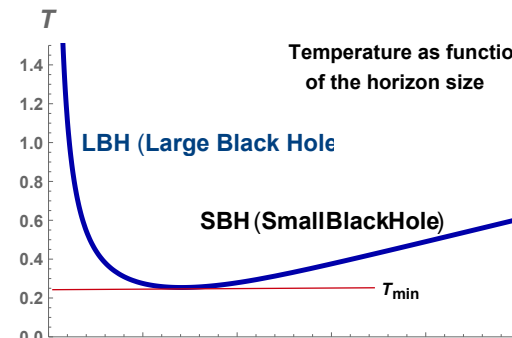
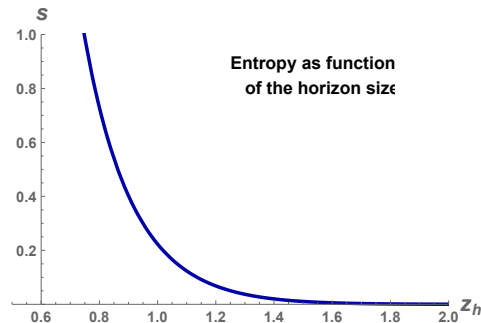
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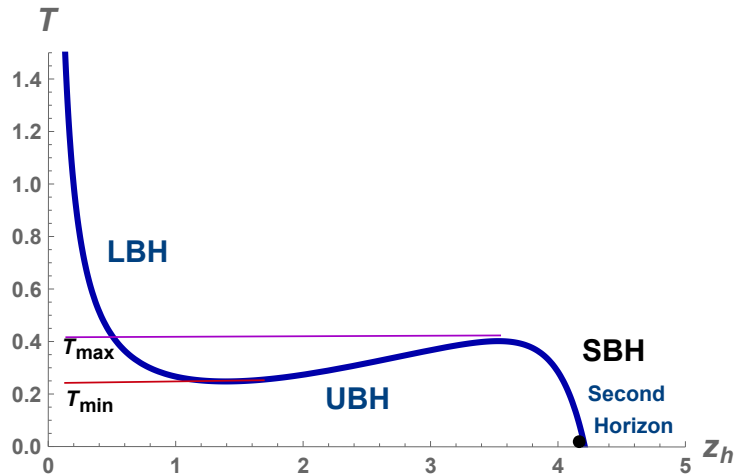


Entropy as function of the temperature

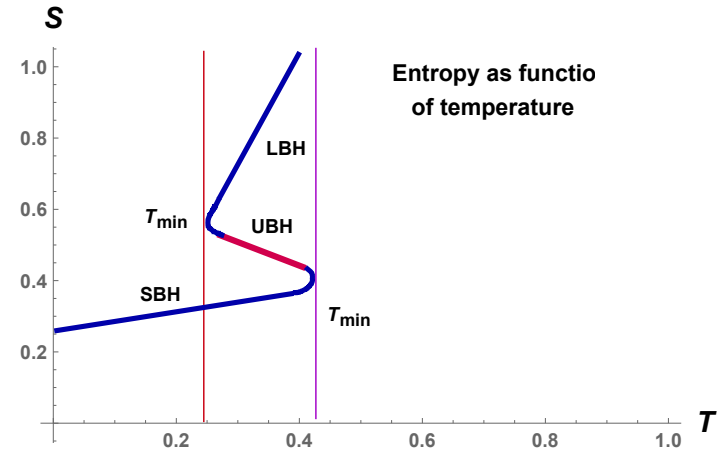
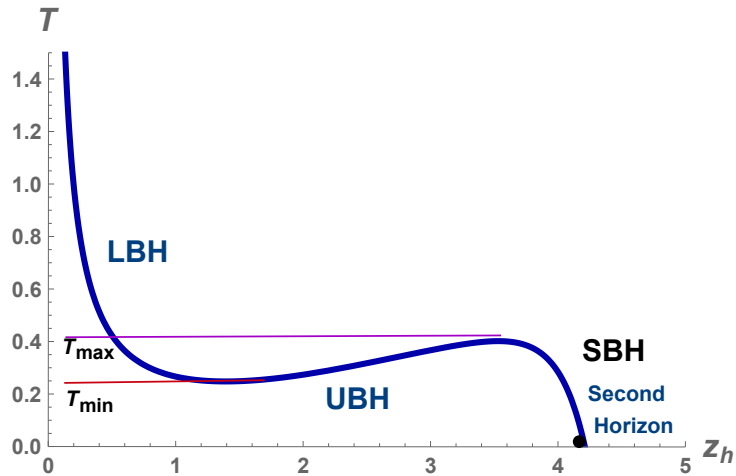
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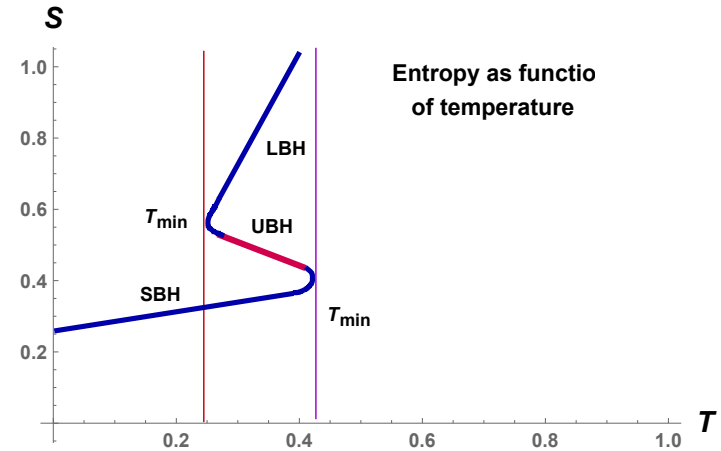
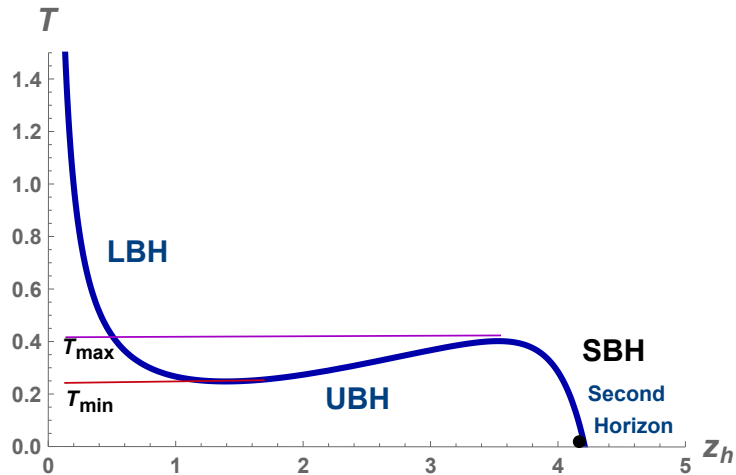
Free energy as function of temperature



Free energy as function of temperature

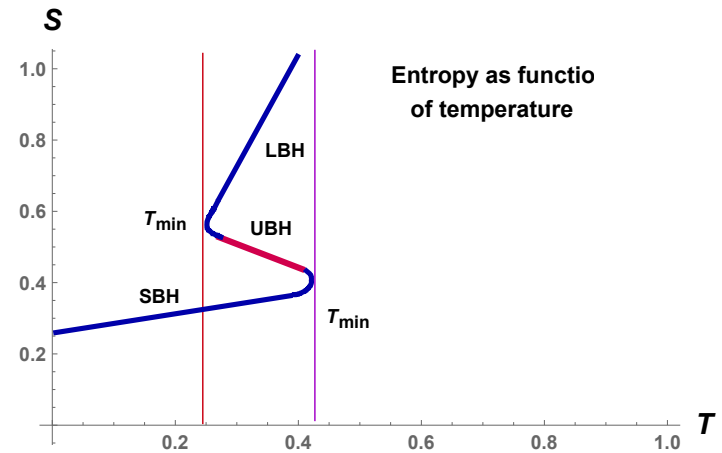
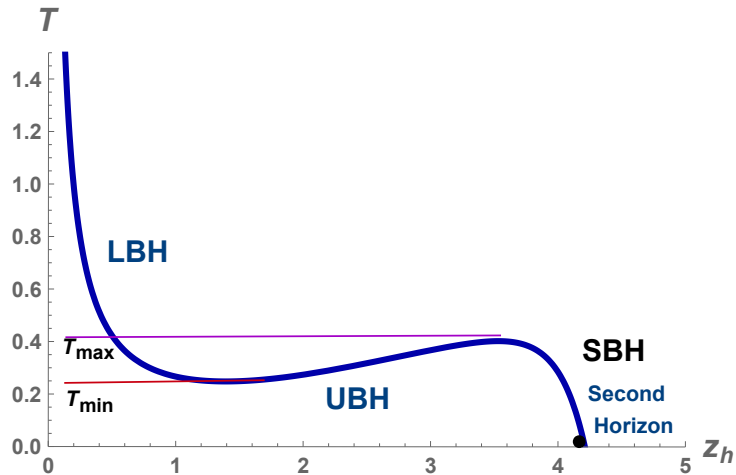


Free energy as function of temperature

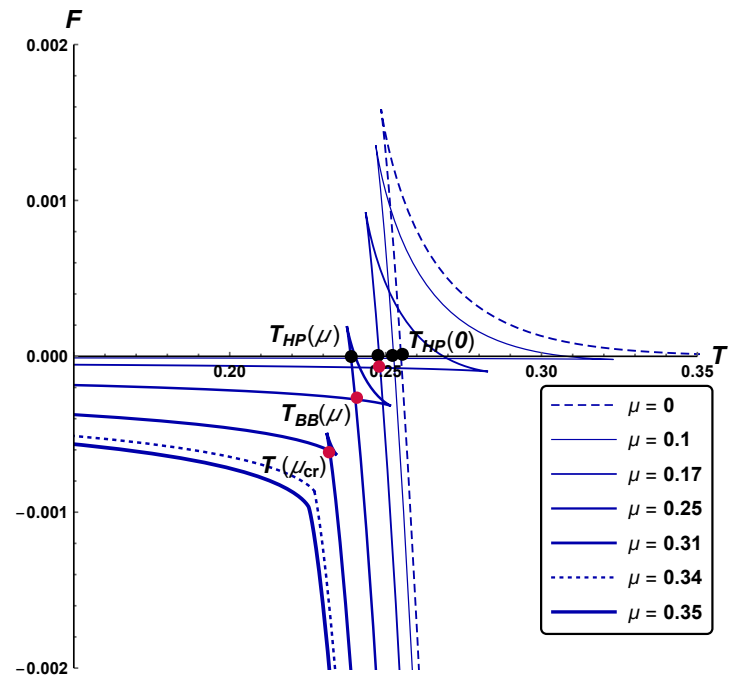


$$F(z_h, c, \nu) = \int_{z_h}^{\infty} s(z_h, c, \nu) T'(z_h, c, \nu) dz_h$$

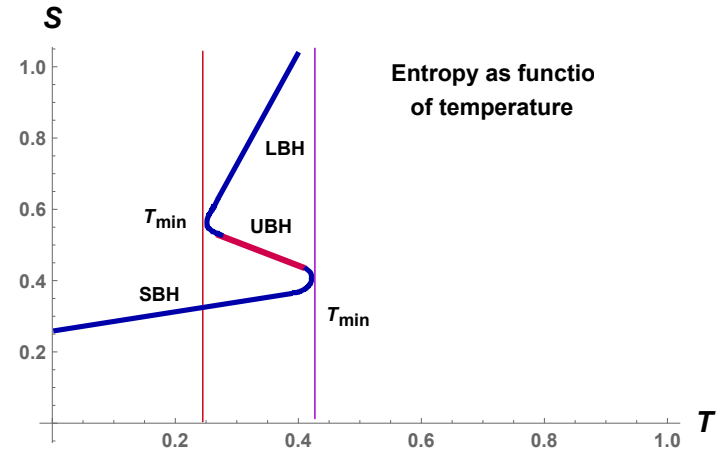
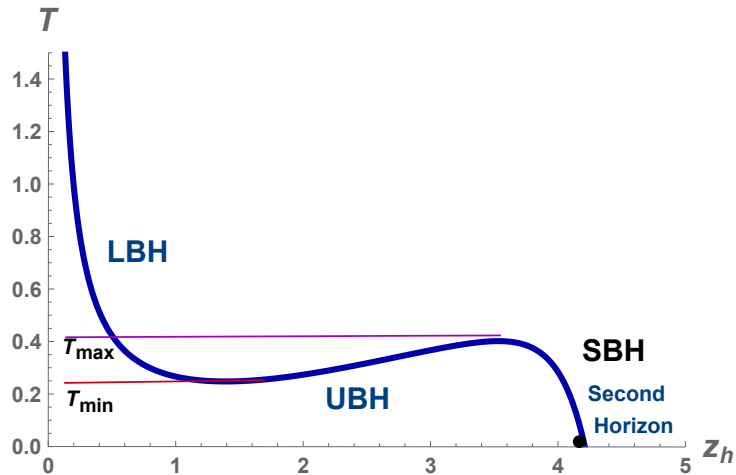
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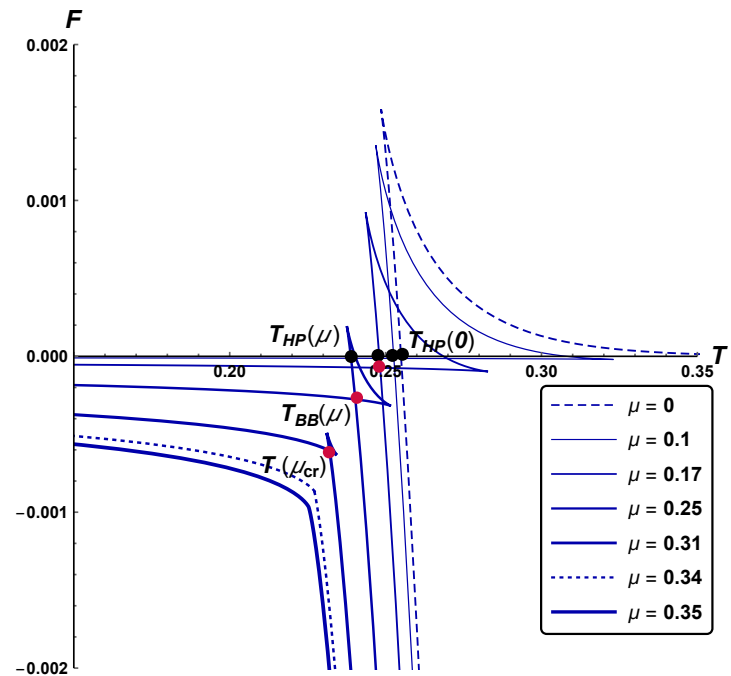


Free energy as function of temperature

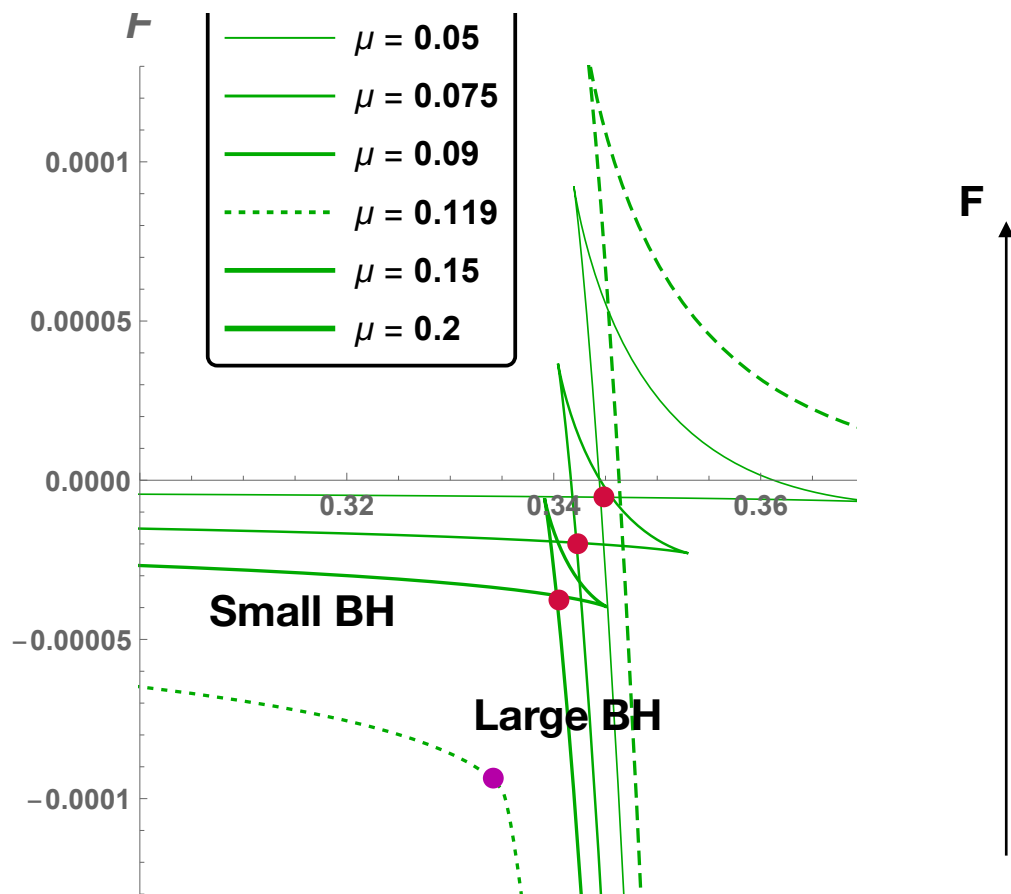


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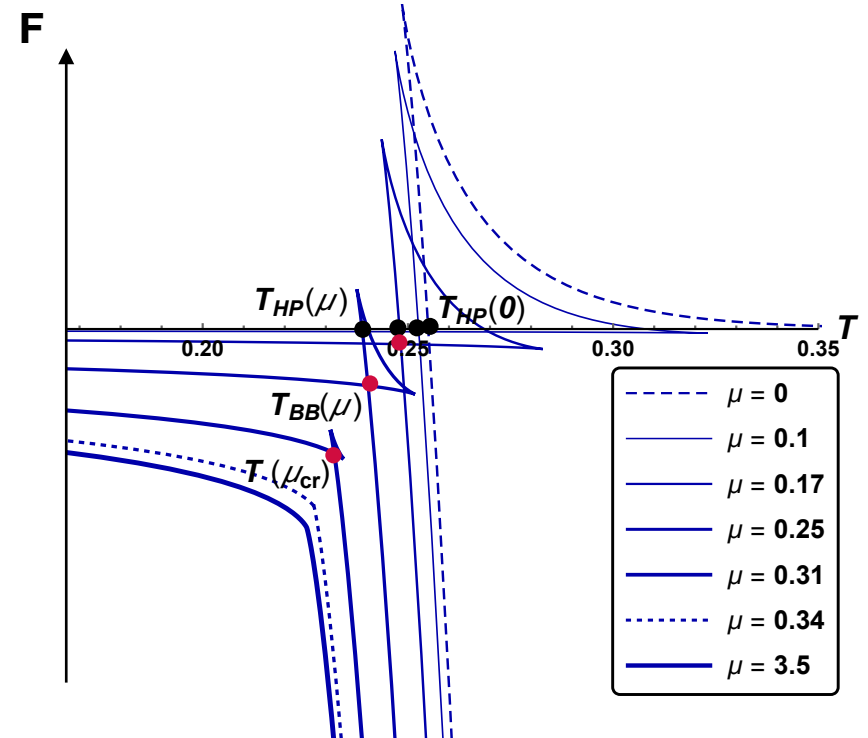
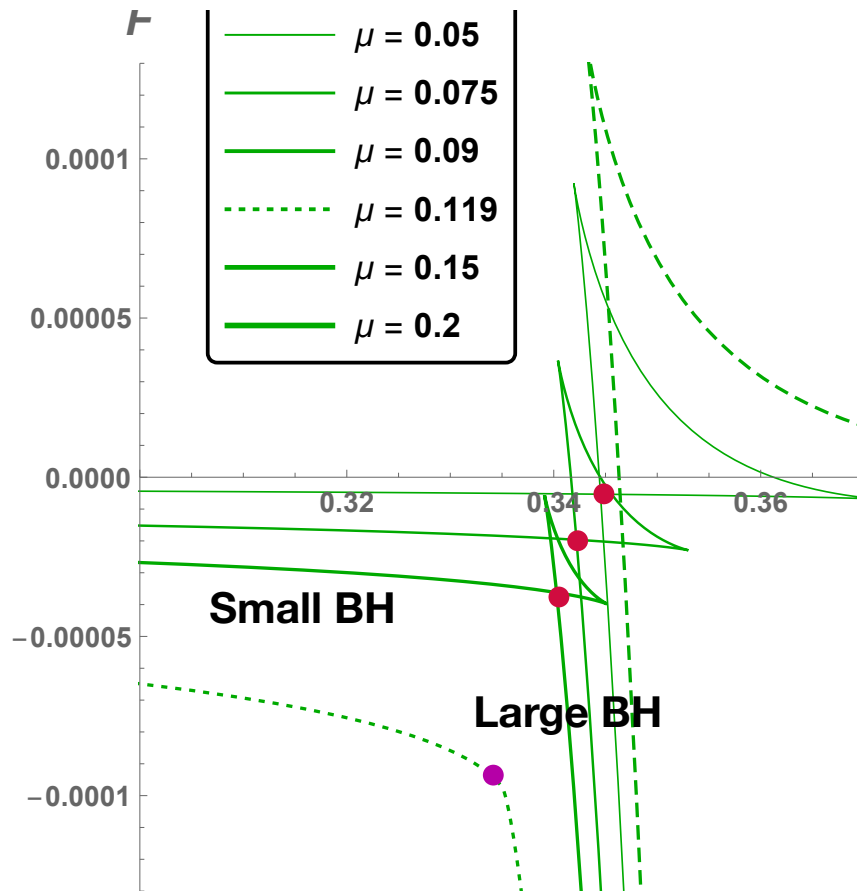
The swallow-tailed shape



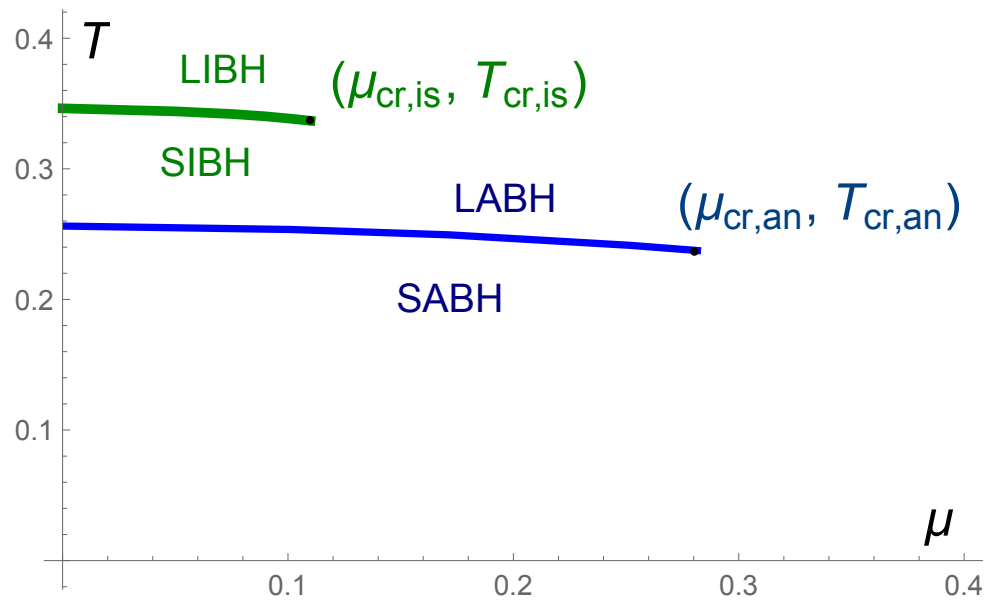
Free energy as function of temperature for **Isotropic** and **Anisotropic**



Free energy as function of temperature for **Isotropic** and **Anisotropic**

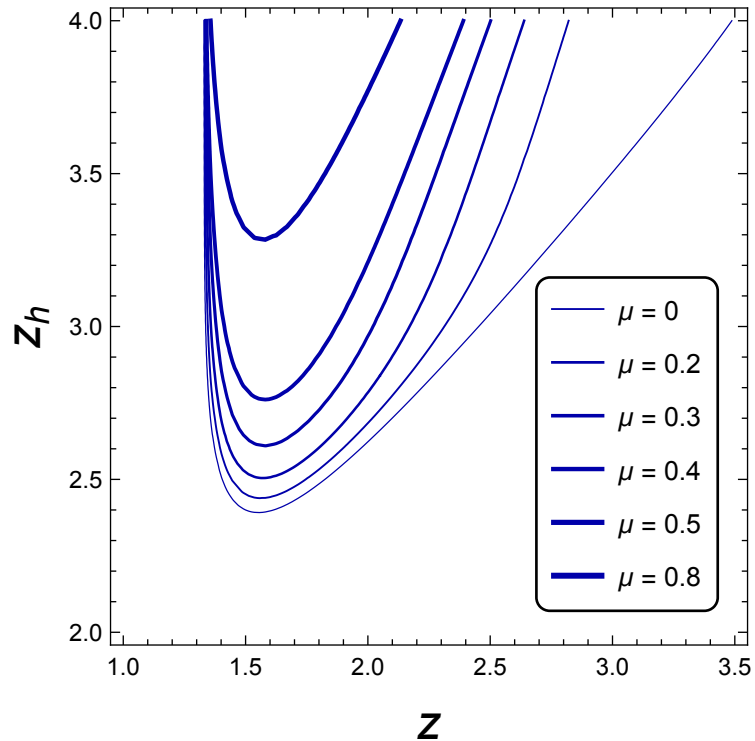


Thermodynamics of the **Anisotropic** background as compare with **Isotropic** one



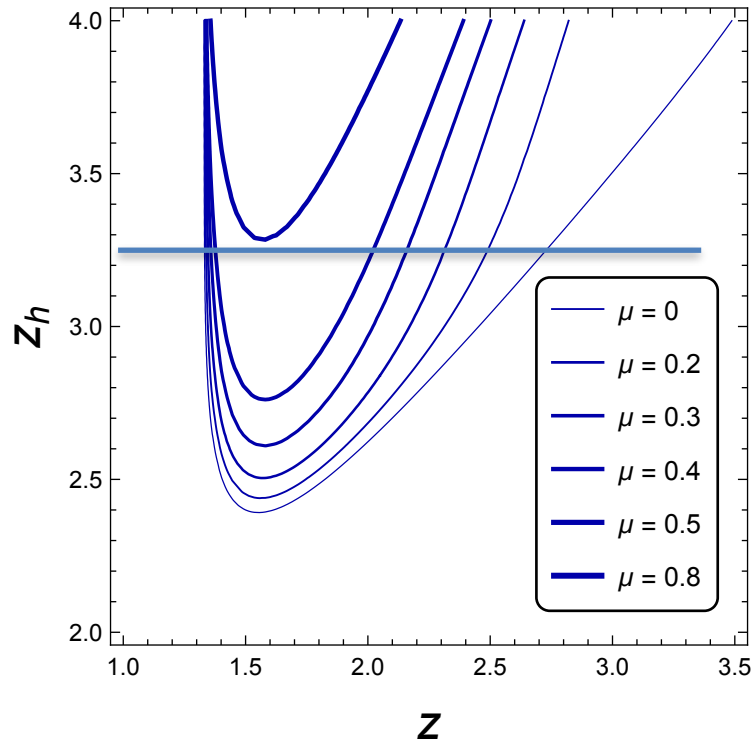
Background instability leads to 1-order confinement/deconfinement phase transition

Dynamical domain wall position



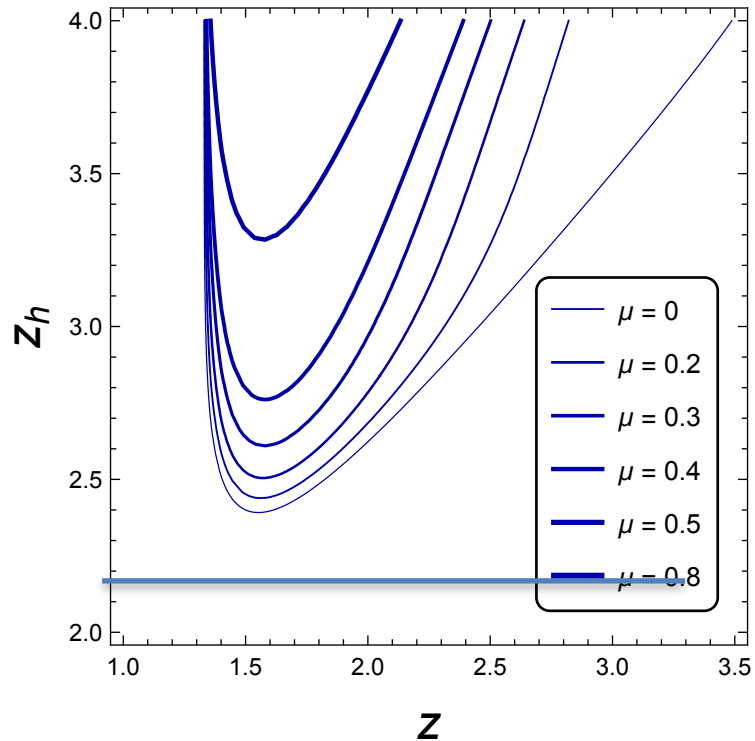
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Dynamical domain wall position



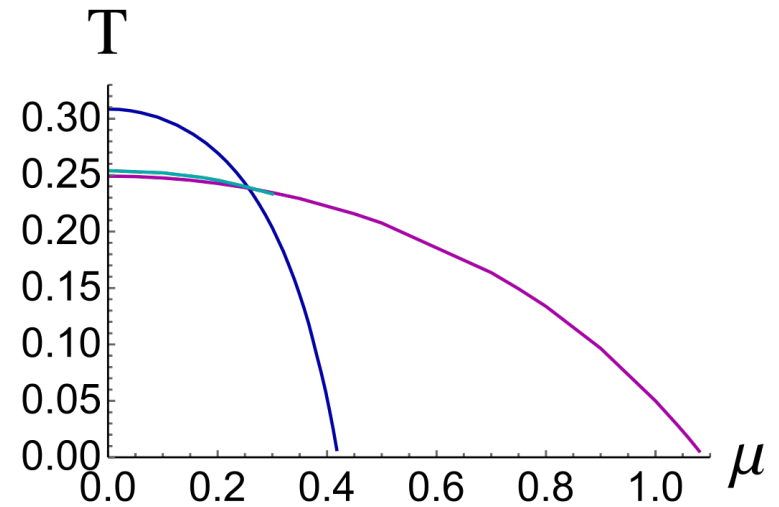
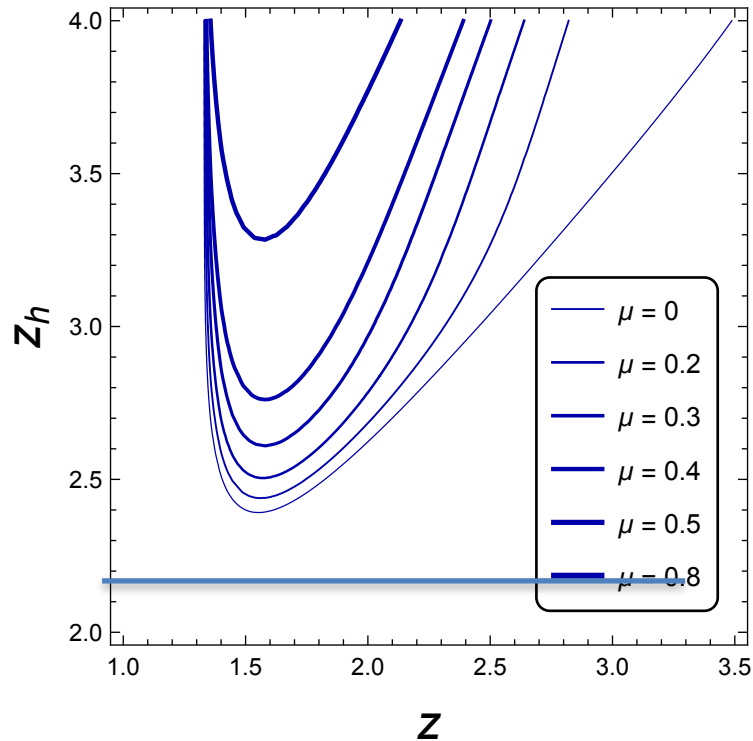
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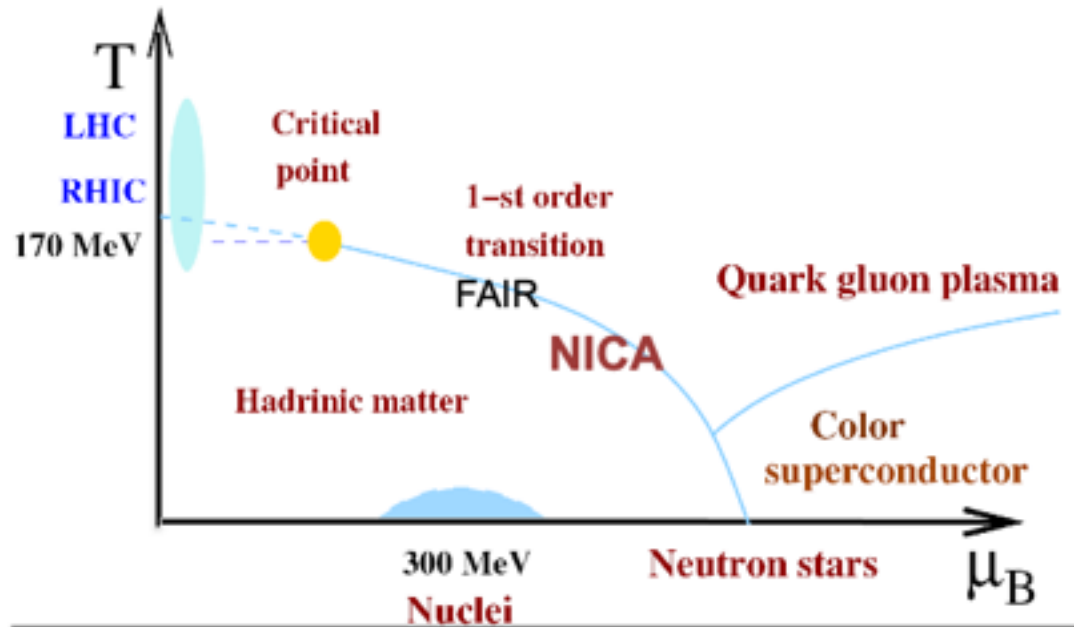
Background instability leads to 1-order confinement/deconfinement phase transition

Dynamical domain wall position

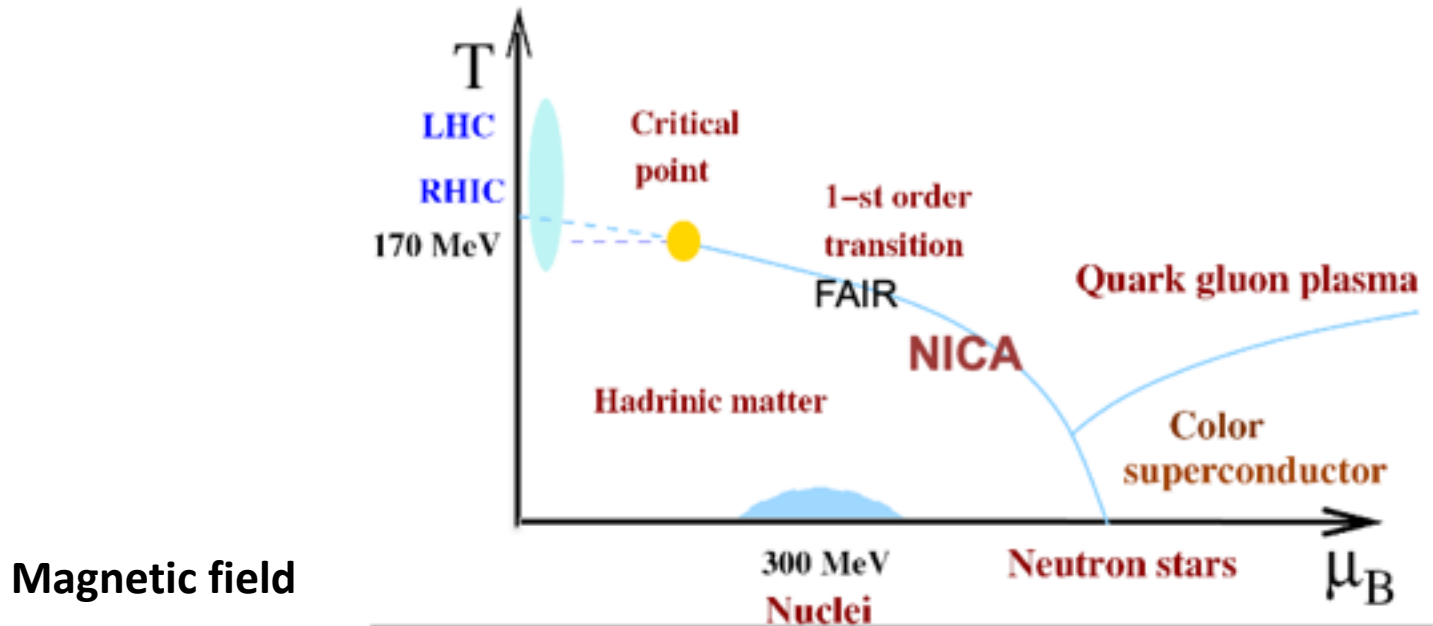


HQCD phase diagram in magnetic field

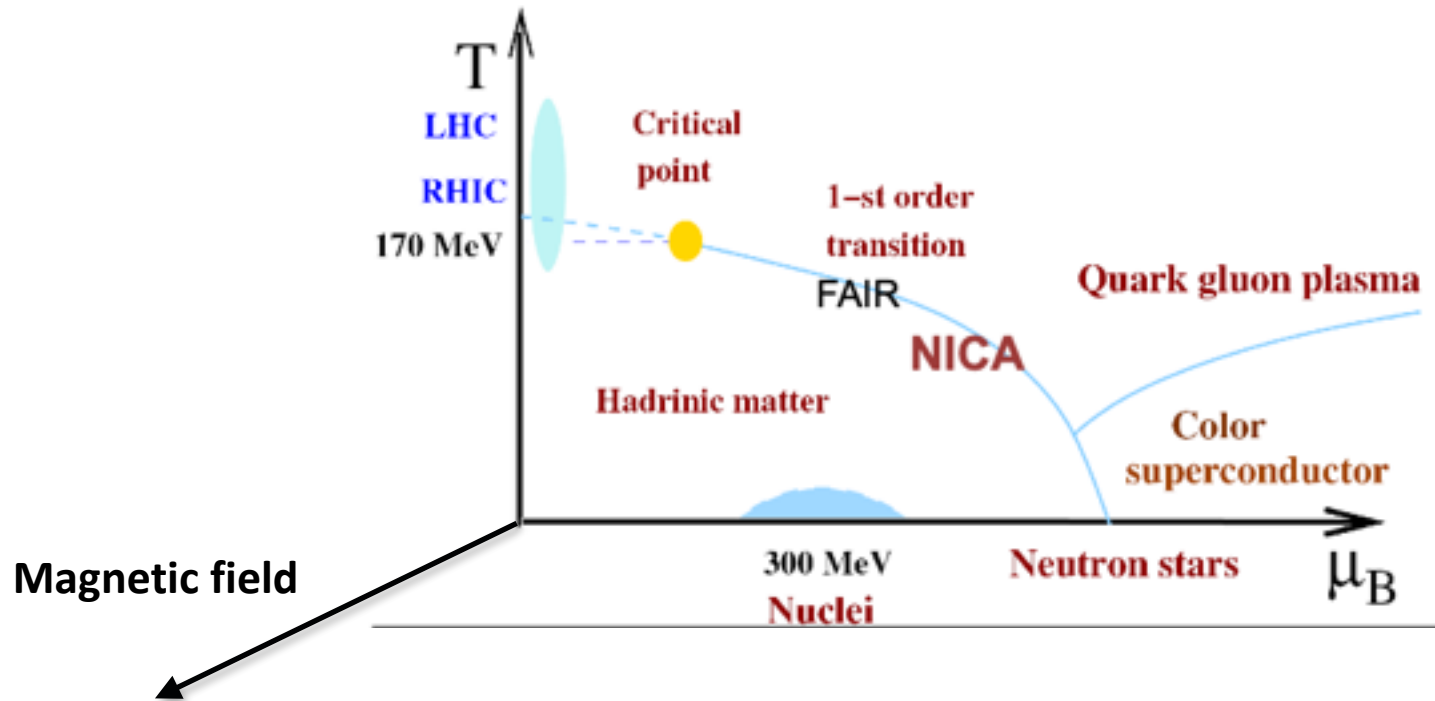
HQCD phase diagram in magnetic field



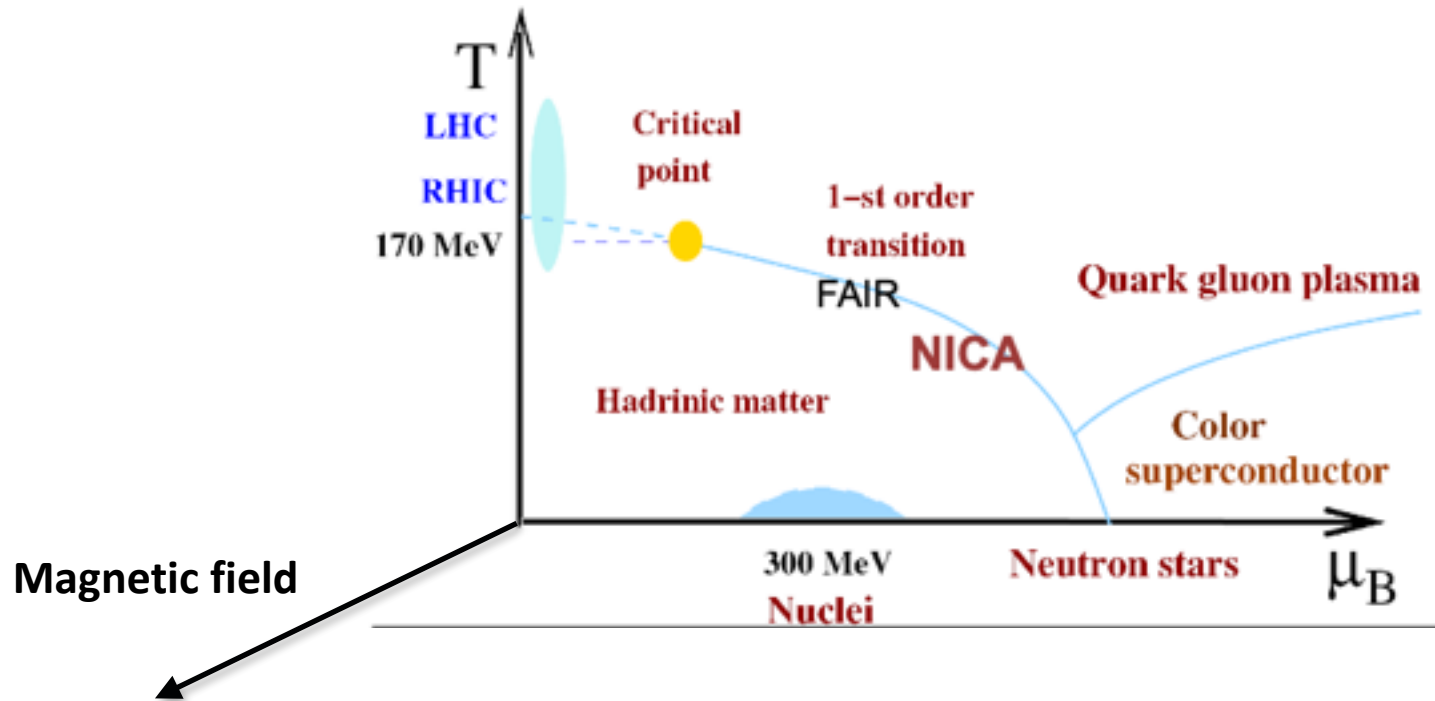
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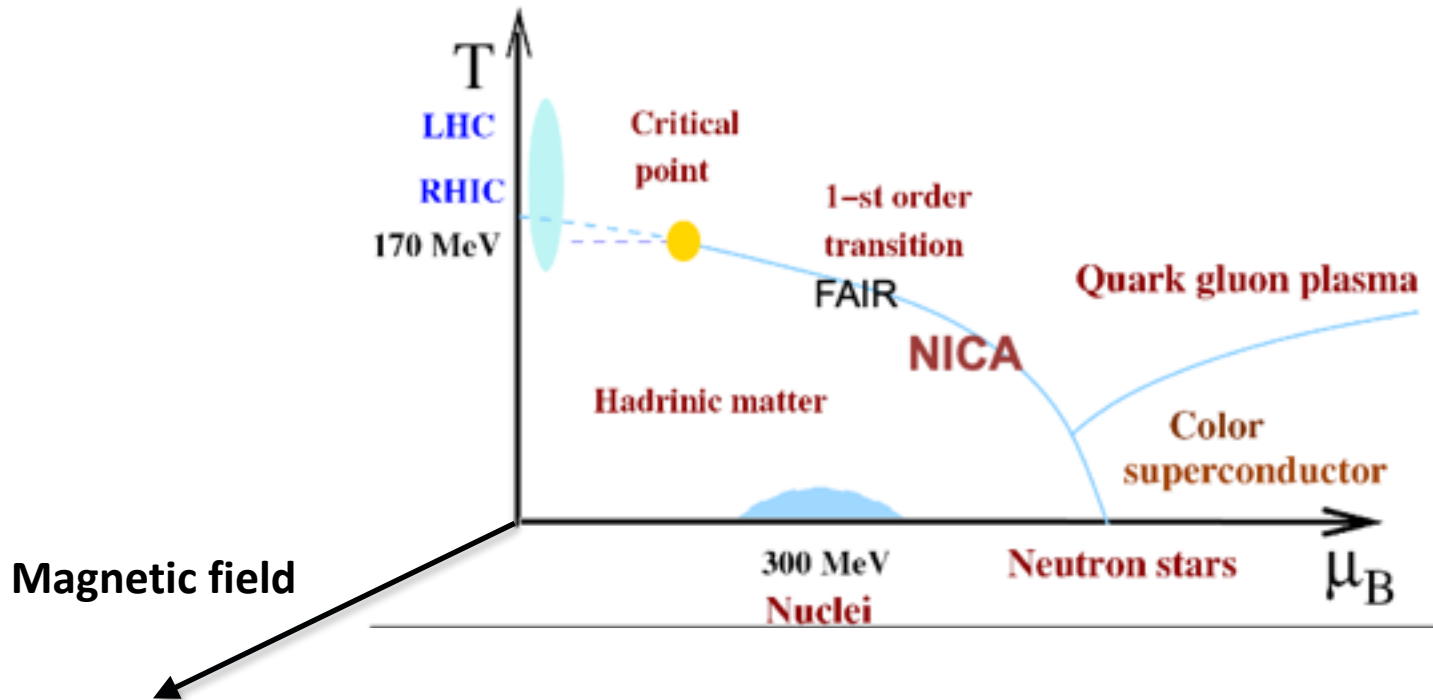


HQCD phase diagram in magnetic field



In HIC magnetic field: a largest known magnitude $\sim 10^{18}$ Gauss

HQCD phase diagram in magnetic field



Lattice data ($\mu=0$) IMC (inverse magnetic catalysis)

In HIC magnetic field: a largest known magnitude $\sim 10^{18}$ Gauss

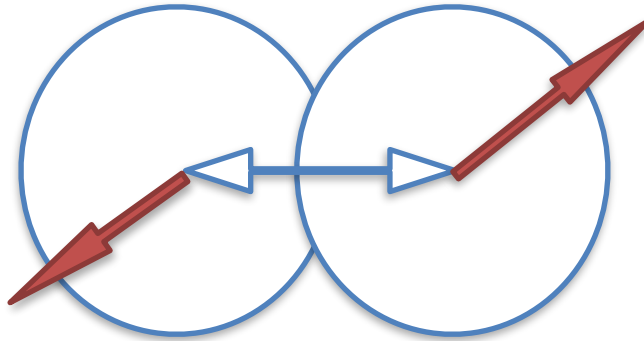
Heavy ion collisions: largest known magnitude $\sim 10^{18}$ Gauss

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Peripheral HIC

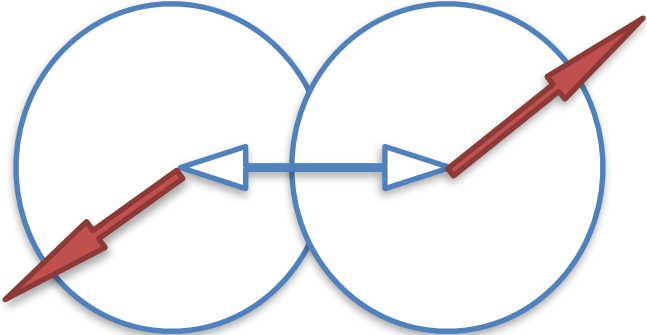
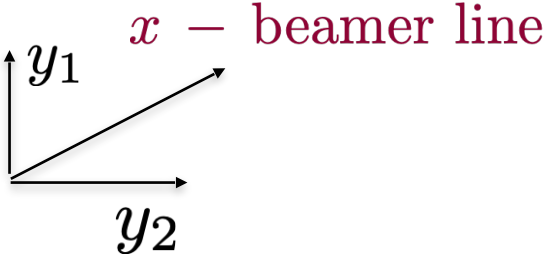
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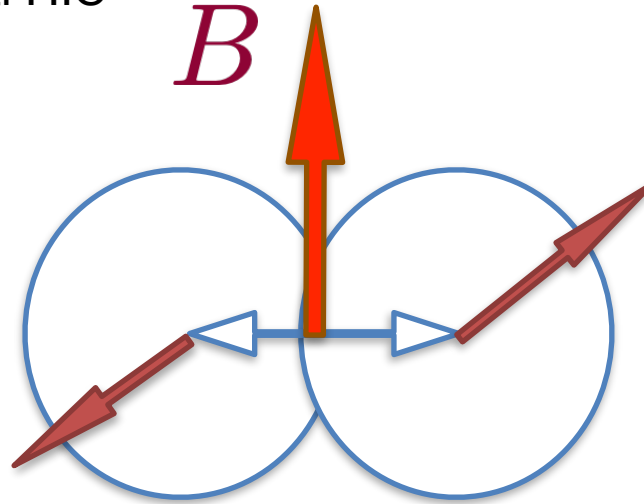
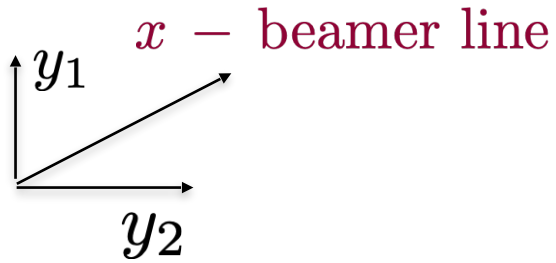
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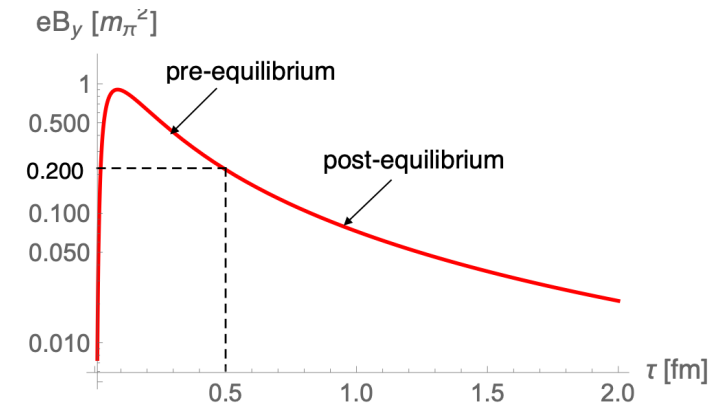
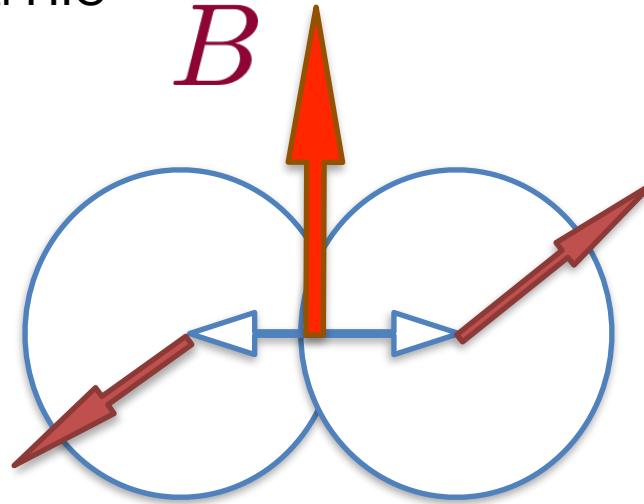
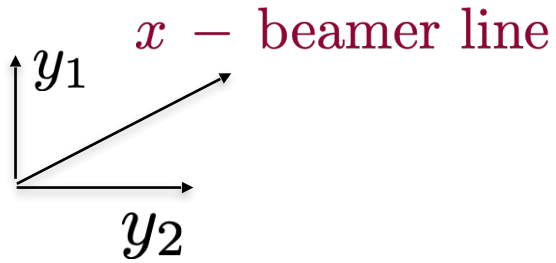
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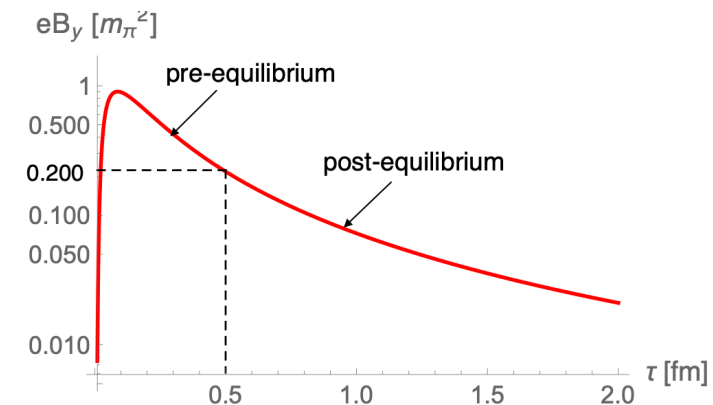
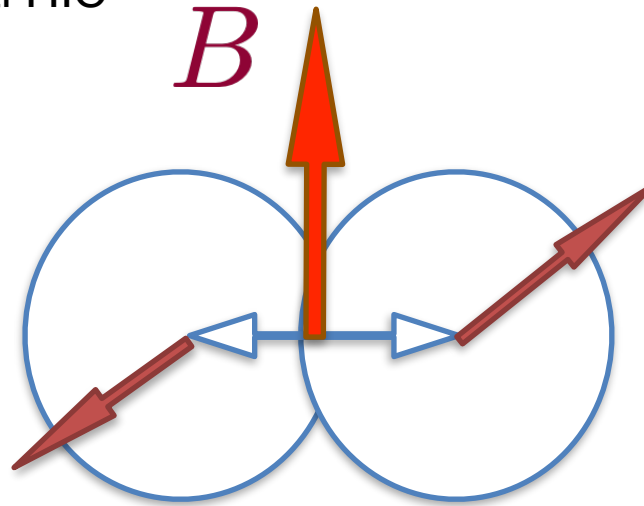
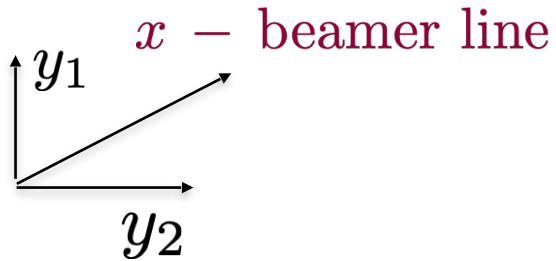
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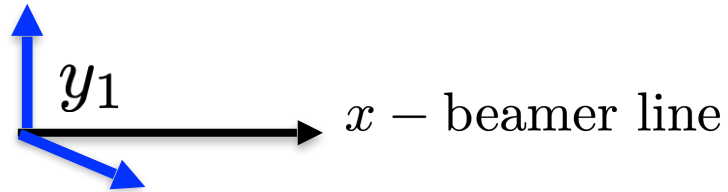


Heavy ion collisions: largest known magnitude $\sim 10^{18}$ Gauss

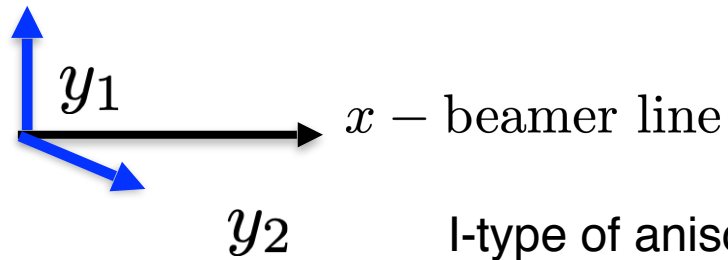
Peripheral HIC



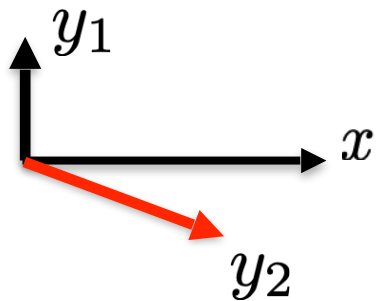
V.Skokov, et al., 0907.1396
V.Tonev, O.Rogachevsky, et al., 1604.0623



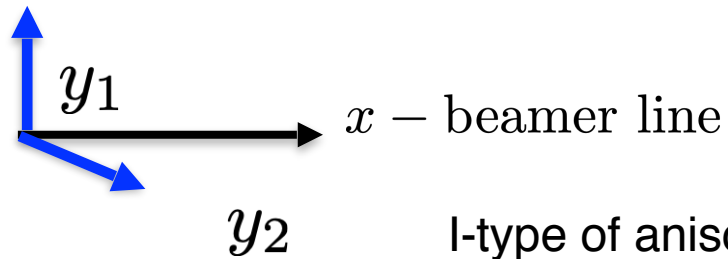
y_2 I-type of anisotropy (transverse to beam vs. along beam)



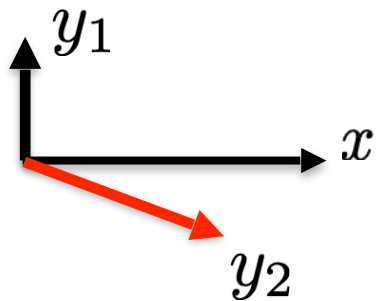
I-type of anisotropy (transverse to beam vs. along beam)



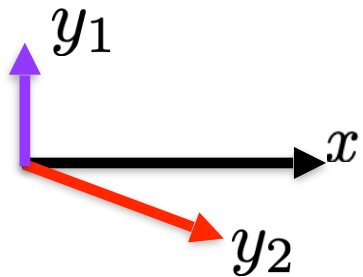
II-type of anisotropy (within y-plane
due to magnetic field)



I-type of anisotropy (transverse to beam vs. along beam)



II-type of anisotropy (within y-plane
due to magnetic field)



III-type of anisotropy (general anisotropy)

5-dim Anisotropic Background at Large Magnetic Field

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Compare with anisotropic model Einstein-Axion-Dilaton action:

$$ds^2 = \frac{L b(z)}{z^2} \left[-g_A(z) dt^2 + dx^2 + dy_1^2 + R_A(z) (dy_2^2) + \frac{dz^2}{g_A(z)} \right]$$

Physical motivations: peripheral HIC

Gursoy, et al
1708.05691,
1811.1172

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IA, K.Rannu, 1802.05652
D.Dudal et al, 1907.01852

Temperature vs horizon

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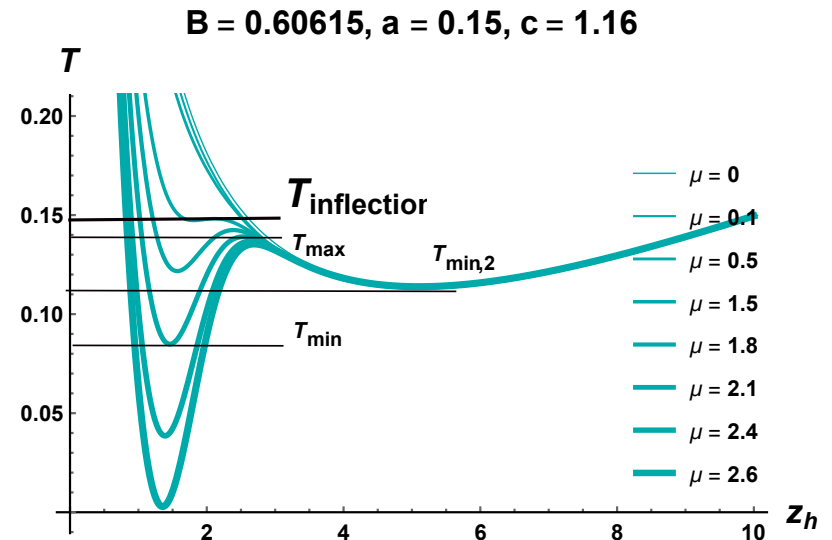
D.Dudal et al, 1907.01852

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D.Dudal et al, 1907.01852

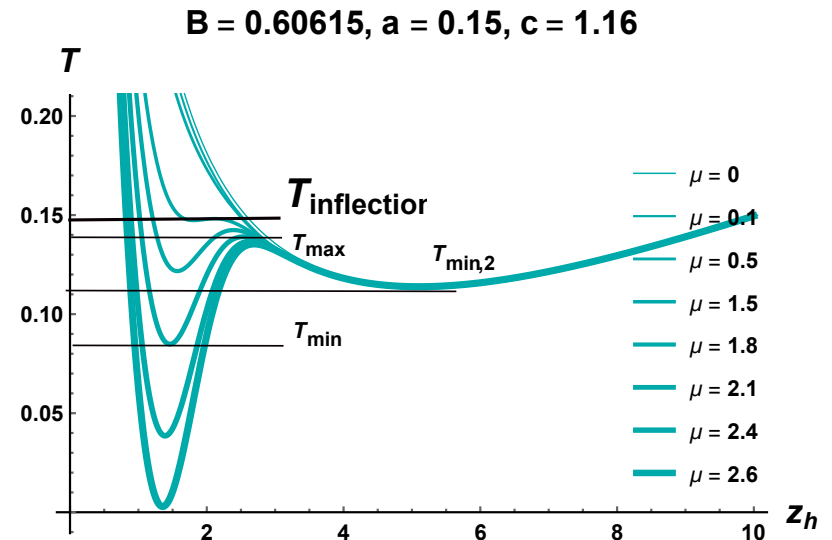
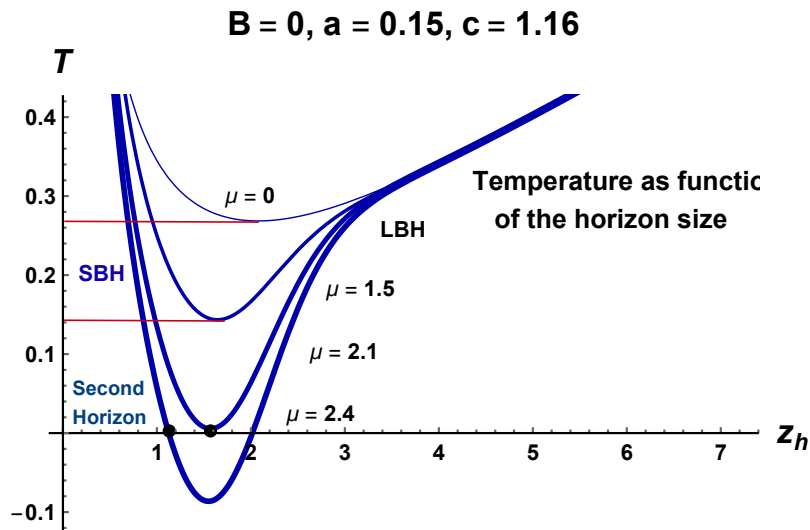


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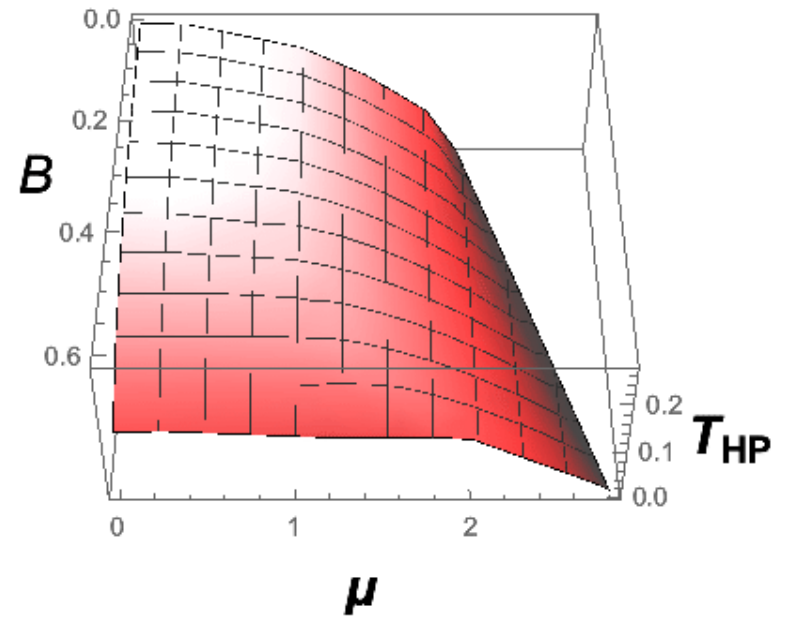
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D.Dudal et al, 1907.01852



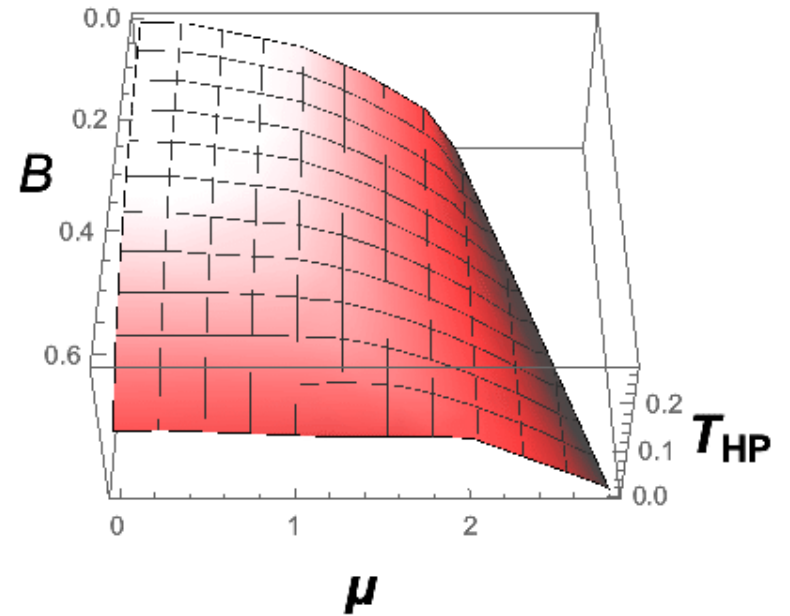
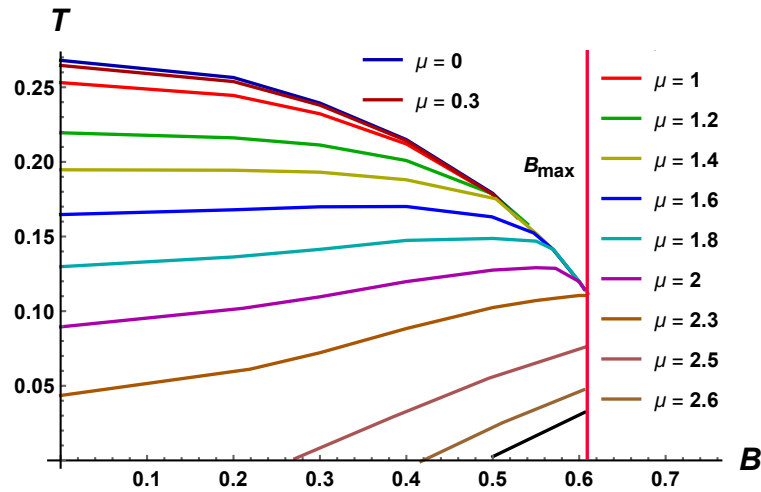
Confinement/deconfinement phase diagram in the magnetic field



D.Dudal et al, 1907.01852

Work in progress:
IA, K.Rannu, P.Slepov

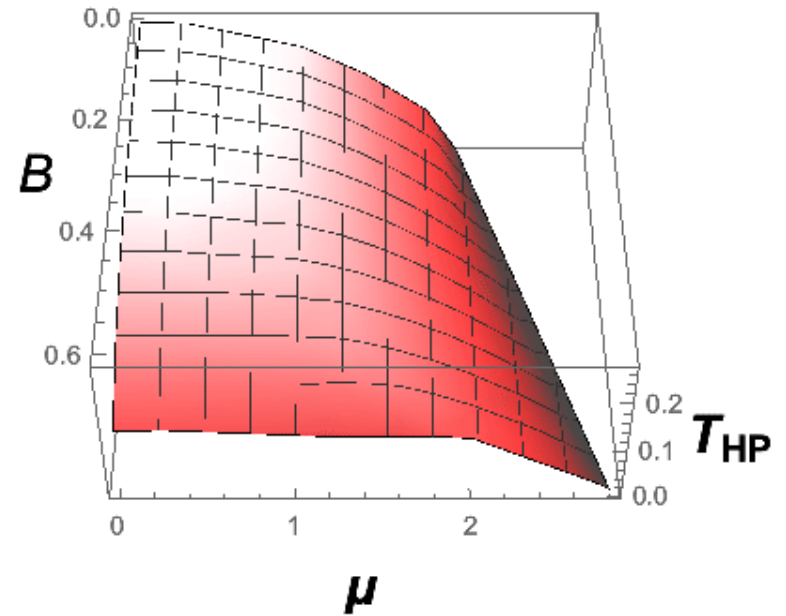
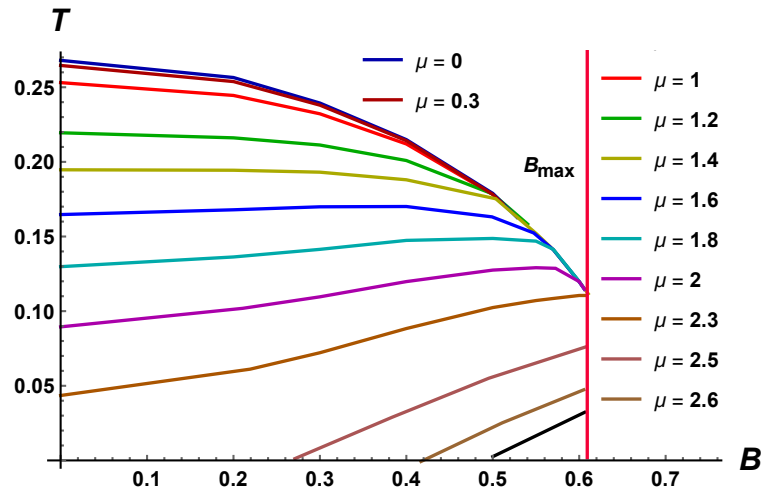
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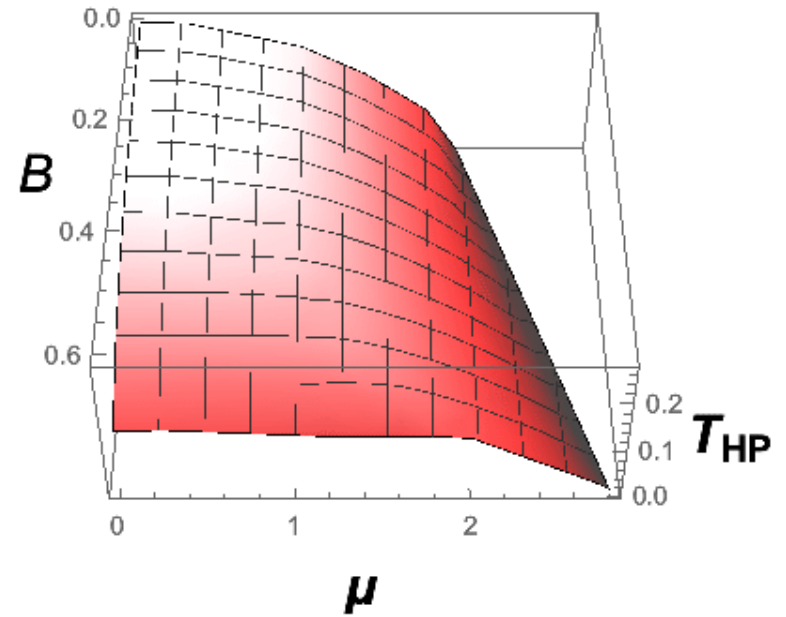
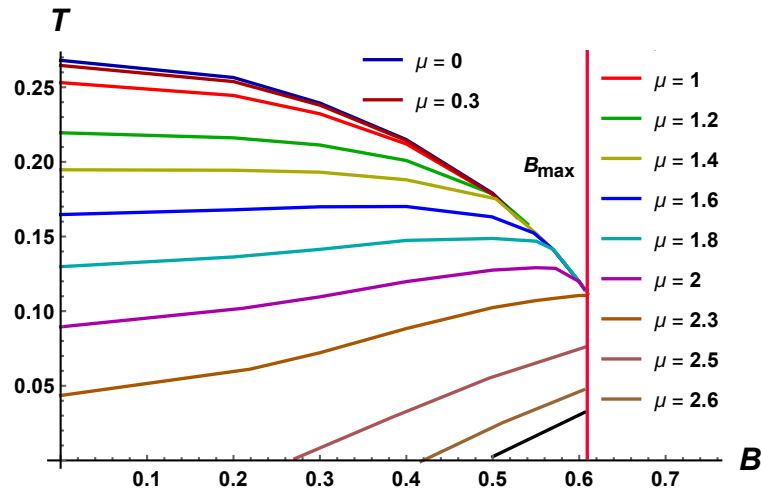
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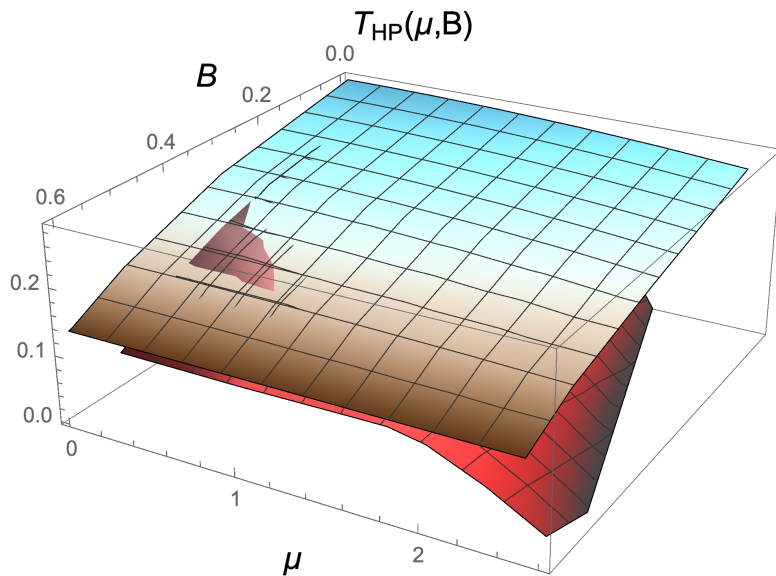
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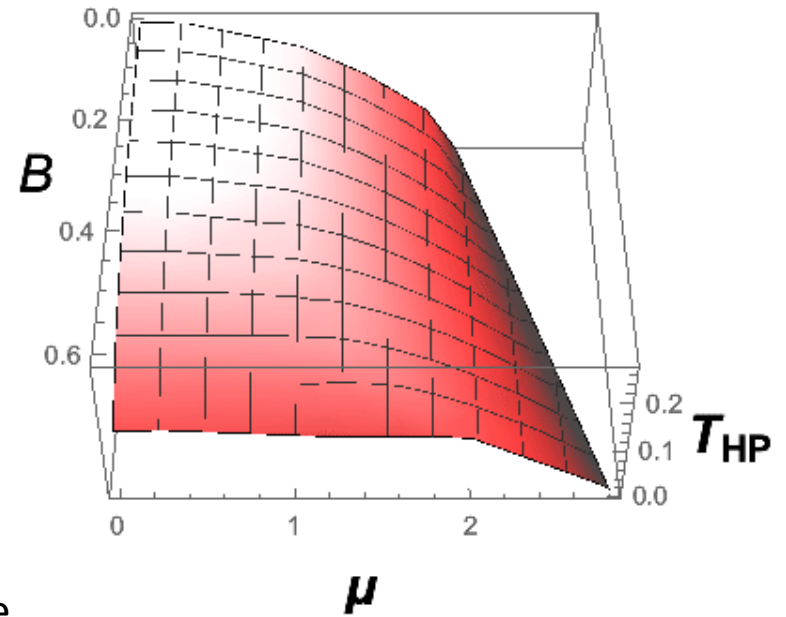
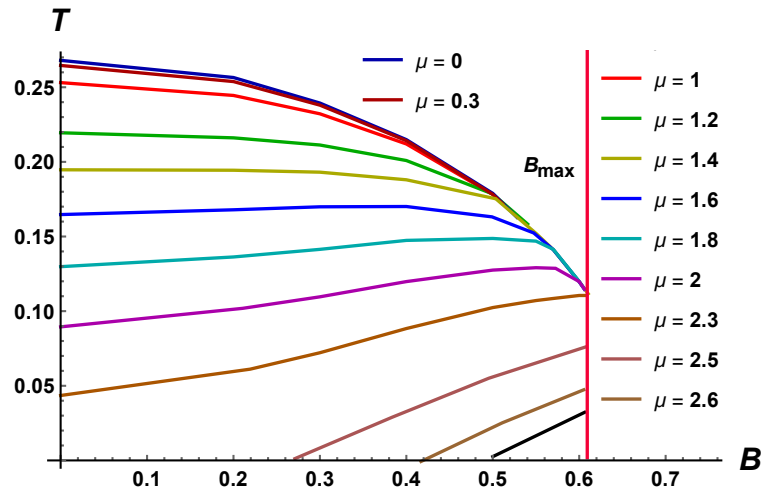


D.Dudal et al, 1907.01852



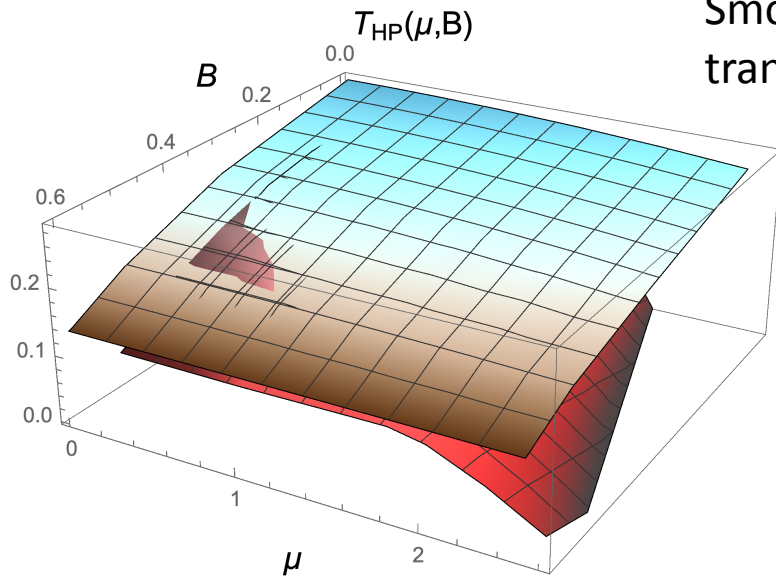
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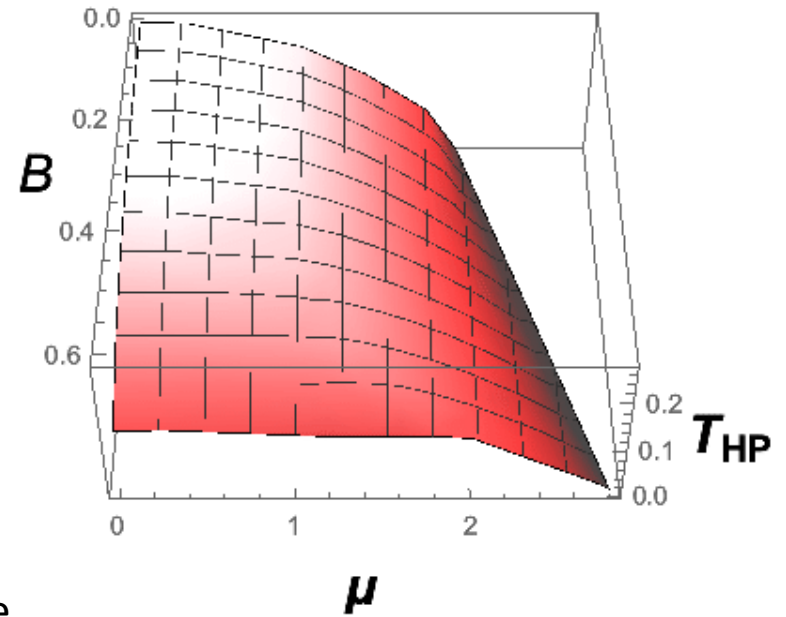
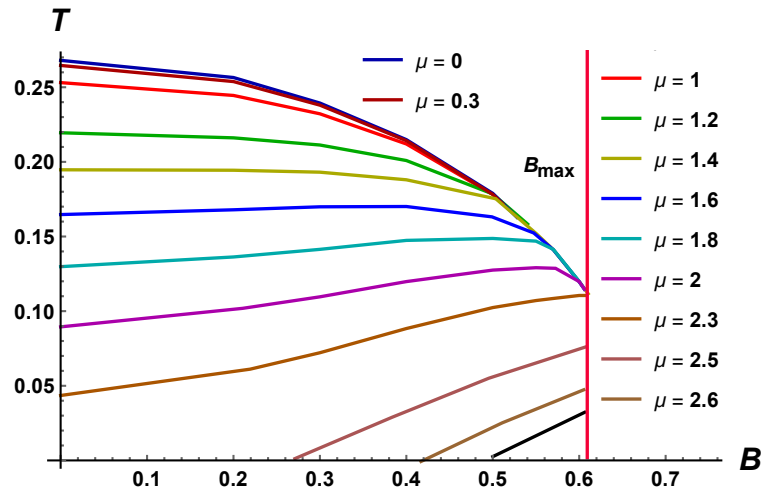
D.Dudal et al, 1907.01852

Smooth phase transition

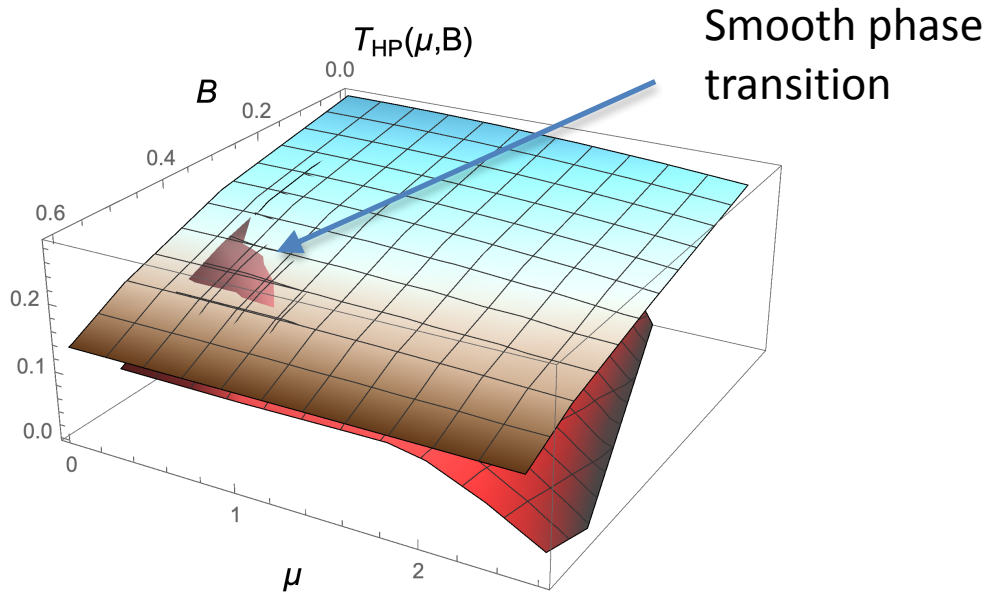


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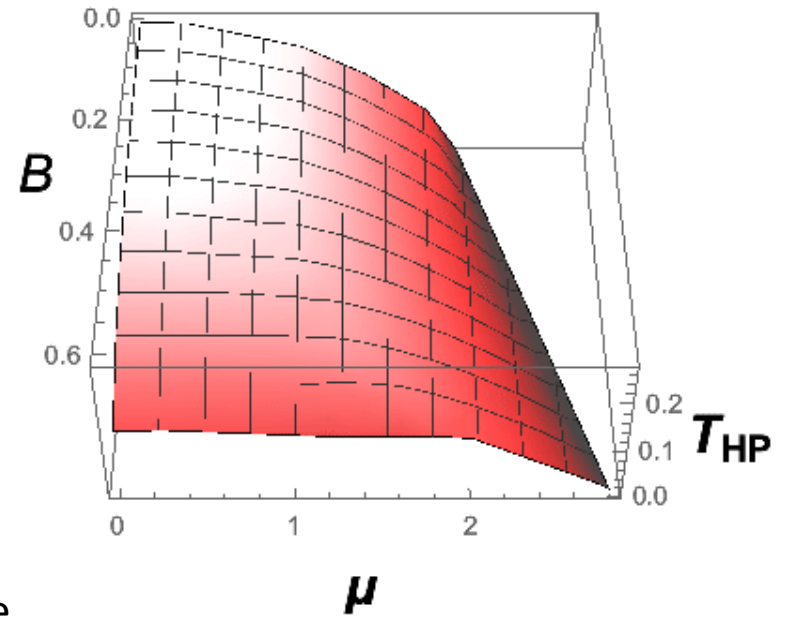
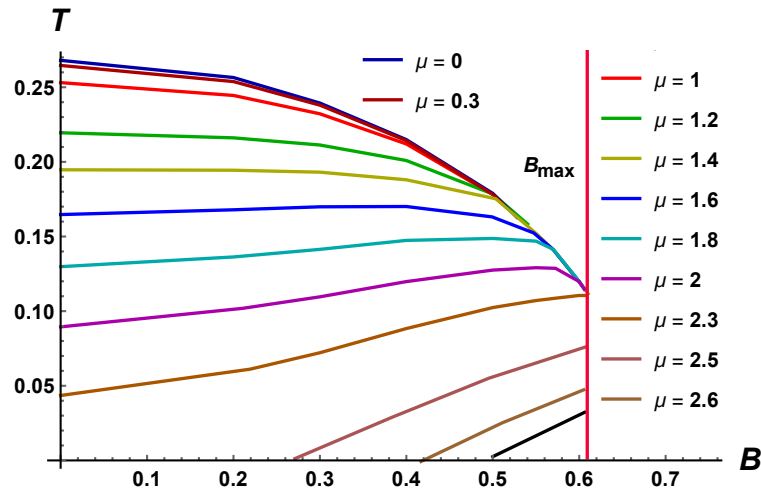


D.Dudal et al, 1907.01852

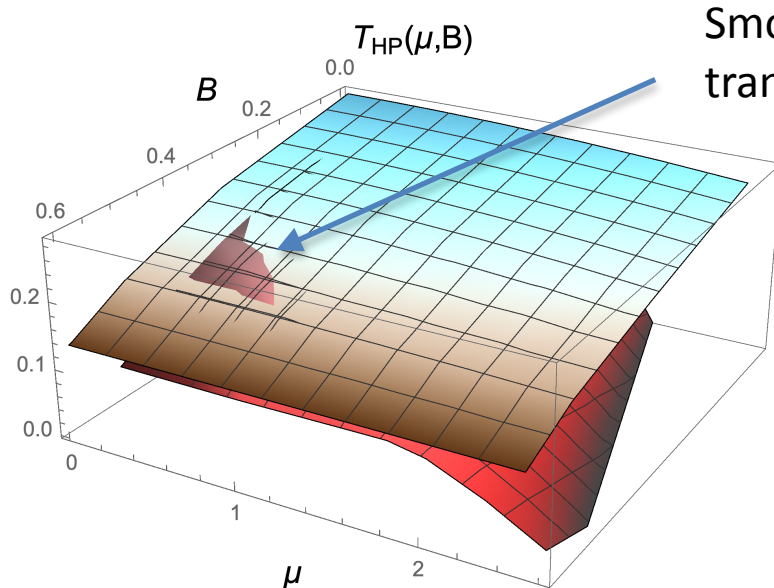


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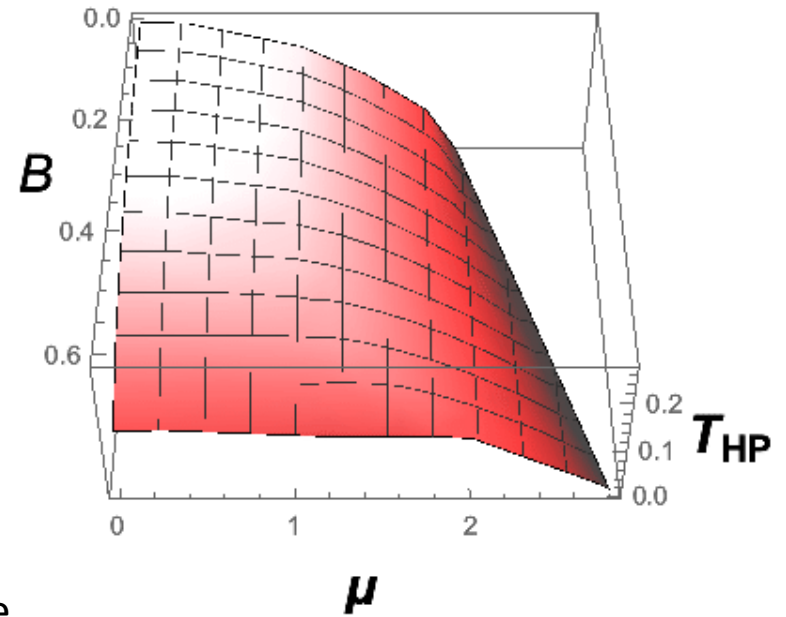
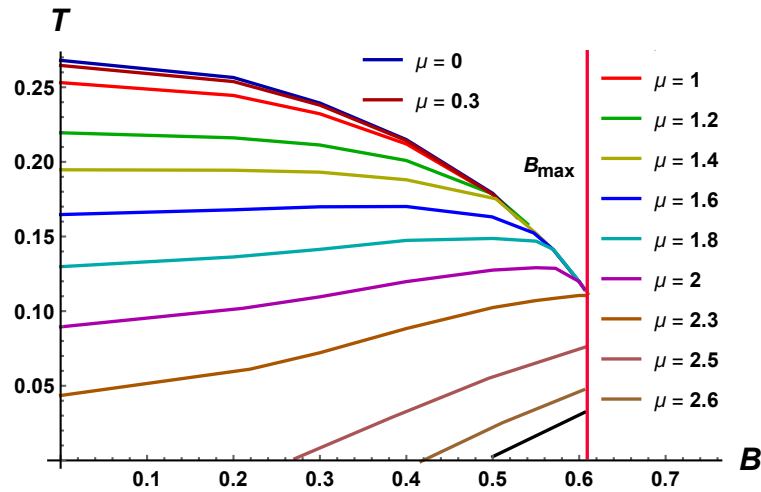


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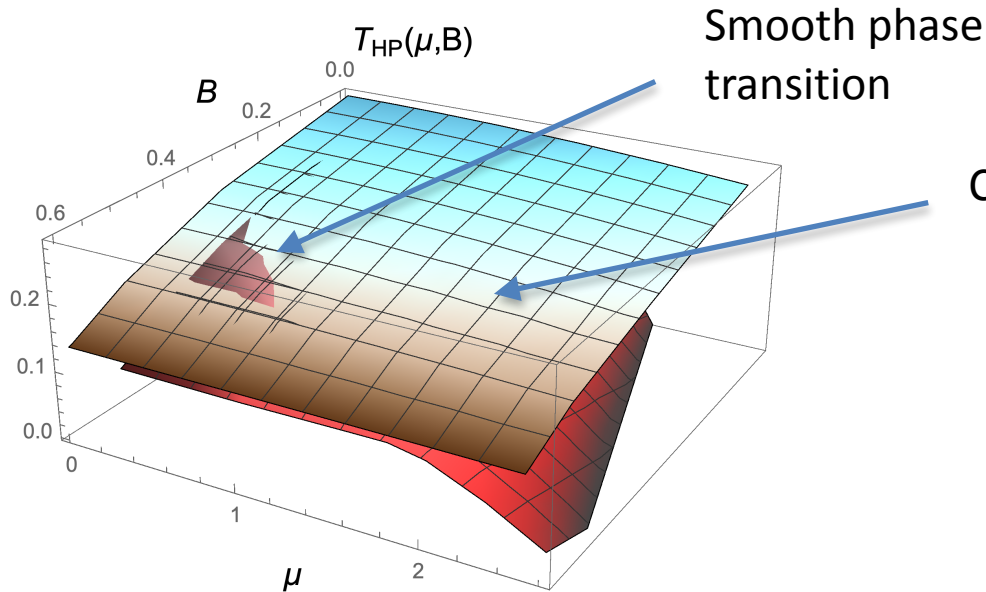
Conf/deconf -> first order

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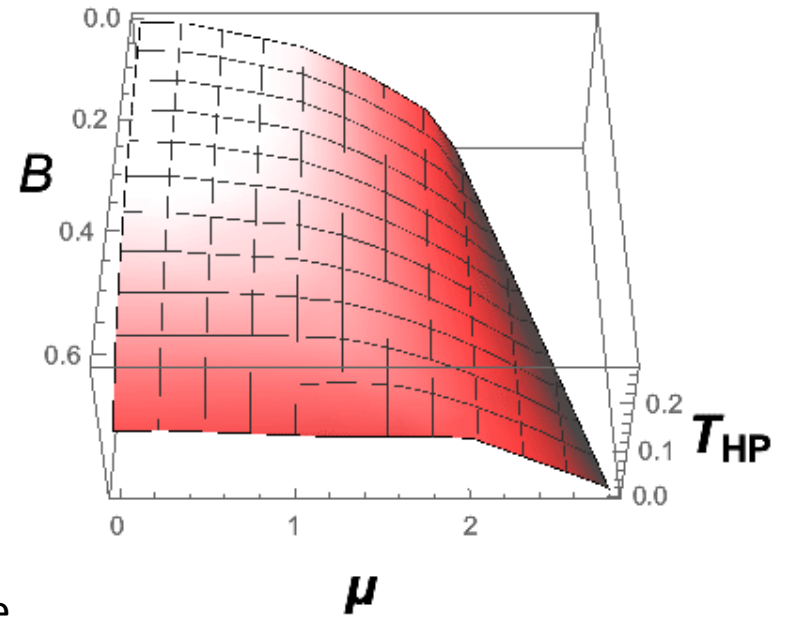
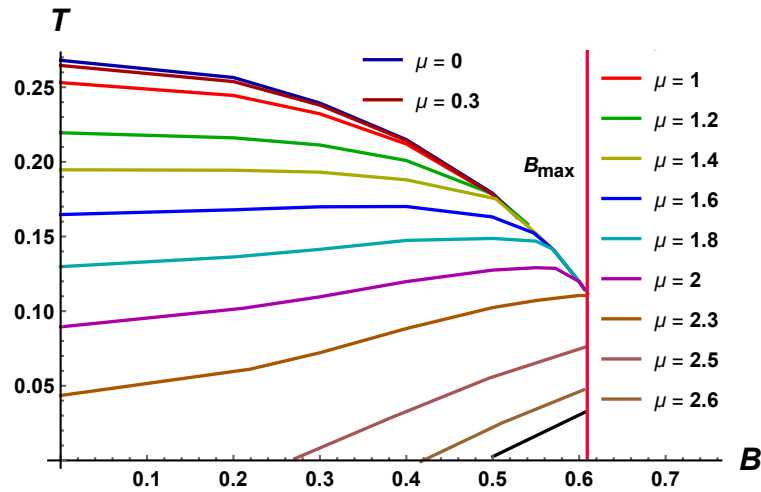


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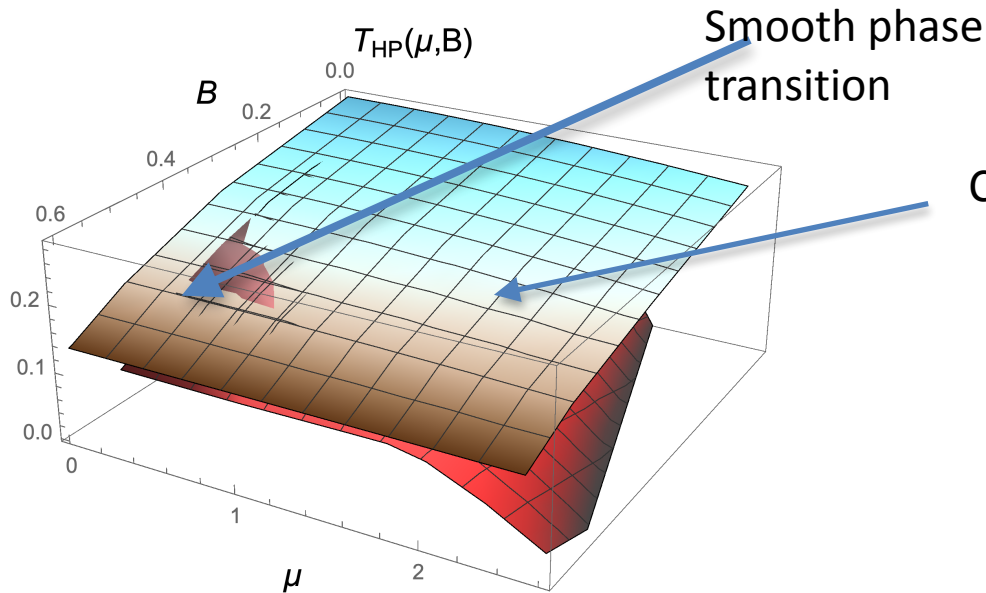


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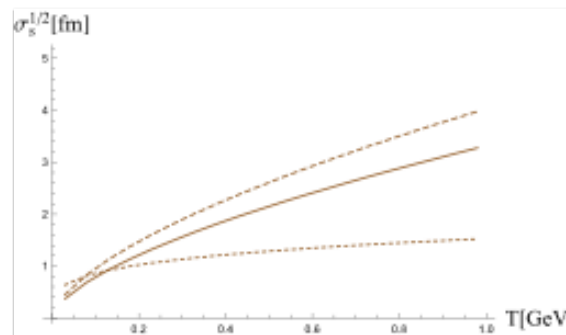
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Work in progress:
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Spatial Wilson loops / Drag Forces / Diffusion coefficients

Gubser, 0605182

$$V_{xY}(z) = \frac{b(z)}{z^2} \sqrt{f(z)}$$

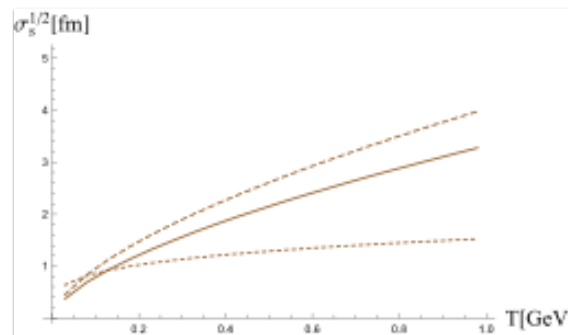
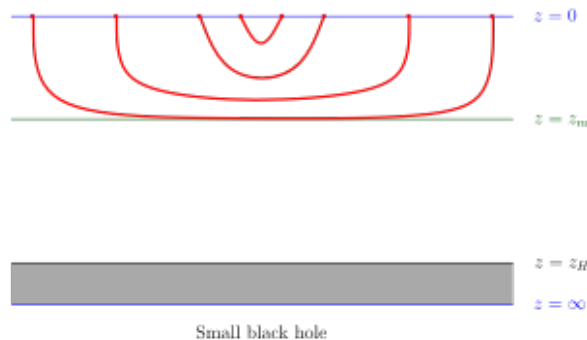


solid - Xy plane
dashed - xY plane,
dotted y_1Y_2 plane.

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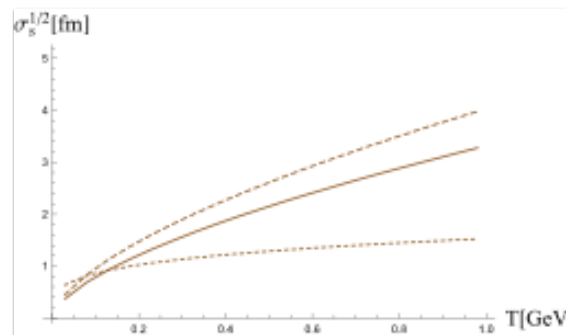
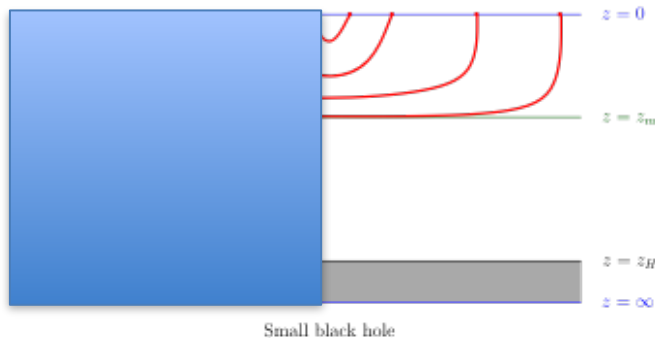


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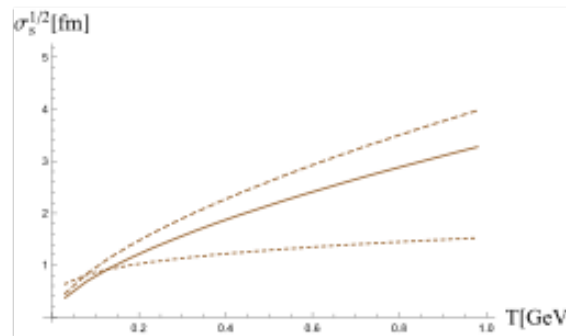
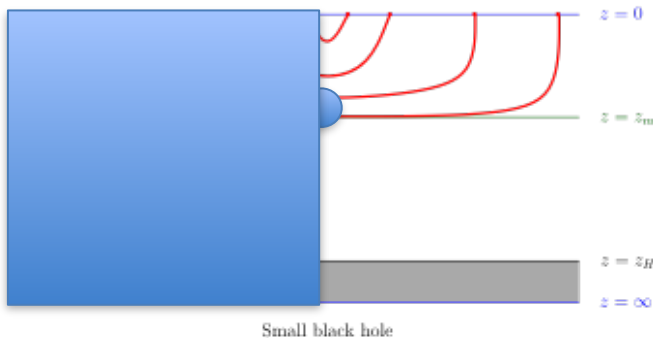


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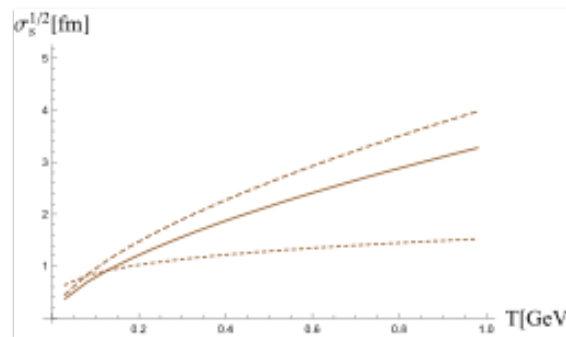
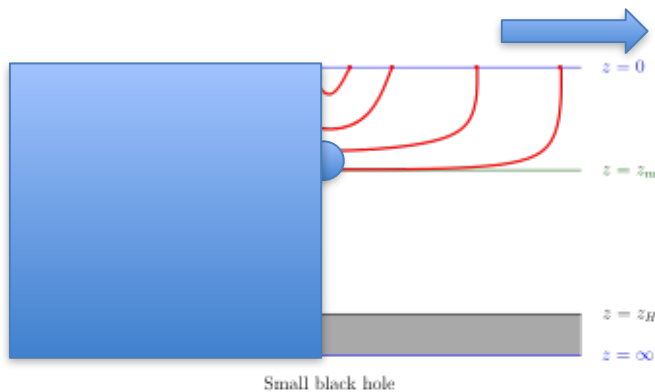


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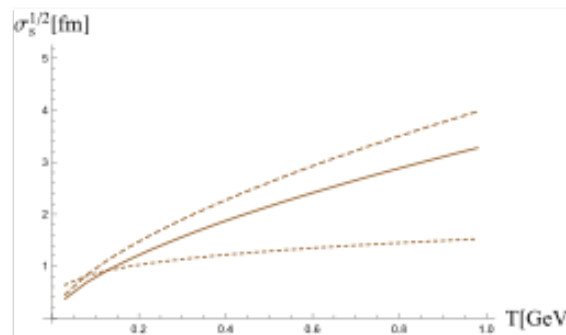
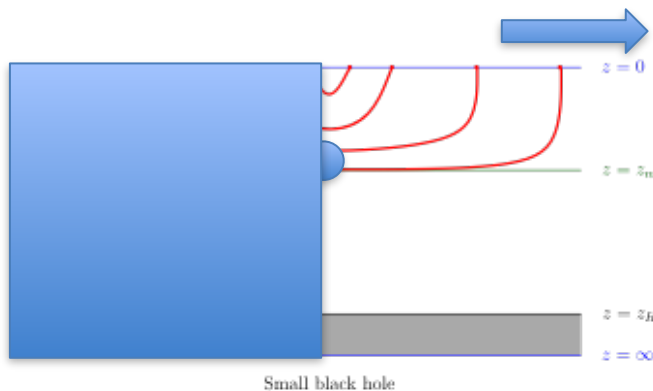
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Spatial Wilson loops / Drag Forces / Diffusion coefficients

$$S_{xY}(\ell)$$

Gubser, 0605182

$$V_{xY}(z) = \frac{b(z)}{z^2} \sqrt{f(z)}$$



solid - Xy plane
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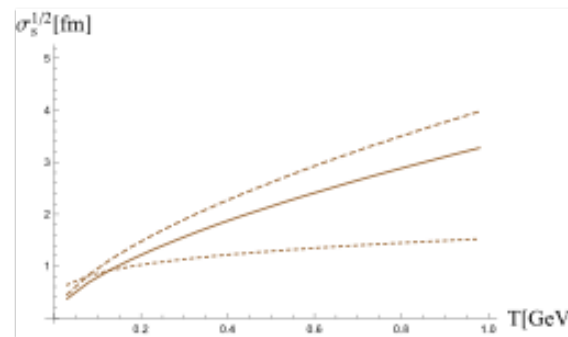
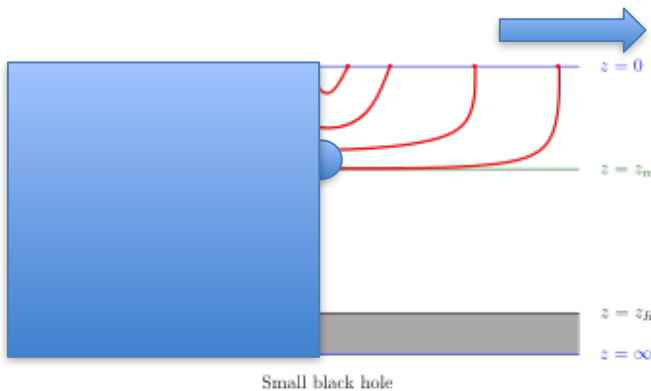
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Gubser, 0605182

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Jet quenching

The jet-quenching parameter is related with the average of the light-like Wilson loop in the adjoint representation

Kovner, Wiedemann, hep/ph 0106240

$$W_A(C) = e^{-\frac{1}{4\sqrt{2}} \hat{q} L_- \ell^2} = e^{2iS_{string}}$$

H. Liu, K. Rajagopal and Wiedemann, PRL'06

C is a rectangular contour with large extension L_- in a light-like and small extension ℓ in a transversal one

For x-light-like ($x_- = t-x$) direction

$$\hat{q} = -\frac{2^{\frac{2}{\nu}+2} \nu^{\frac{\nu+2}{\nu}} \pi^{\frac{2}{\nu}-\frac{1}{2}} \Gamma\left(-\frac{\nu}{2\nu+2}\right) T^{\frac{\nu+2}{\nu}}}{(\nu+1)^{\frac{2(\nu+1)}{\nu}} \Gamma\left(1+\frac{1}{2\nu+2}\right)}$$

Ageev, IA, Golubtsova, Gourgoulhon, NPB'18

Direct photons and electric conductivity

$$G_{\mu\nu}^R(k) = i \int d^4x e^{ik \cdot x} \theta(x^0) \langle [J_\mu^a(x), J_\nu^b(0)] \rangle$$

Direct photons and electric conductivity

The thermal-photon production from the QGP plays an essential role.
Photons after they are produced in HIC almost do not interact with the QGP.
Photons give us the local information on heavy ion collisions.

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The photon-emission rate is related to the **retarded correlator** of currents in momentum space

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The photon-emission rate is related to the **retarded correlator** of currents in momentum space

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Direct photons and electric conductivity

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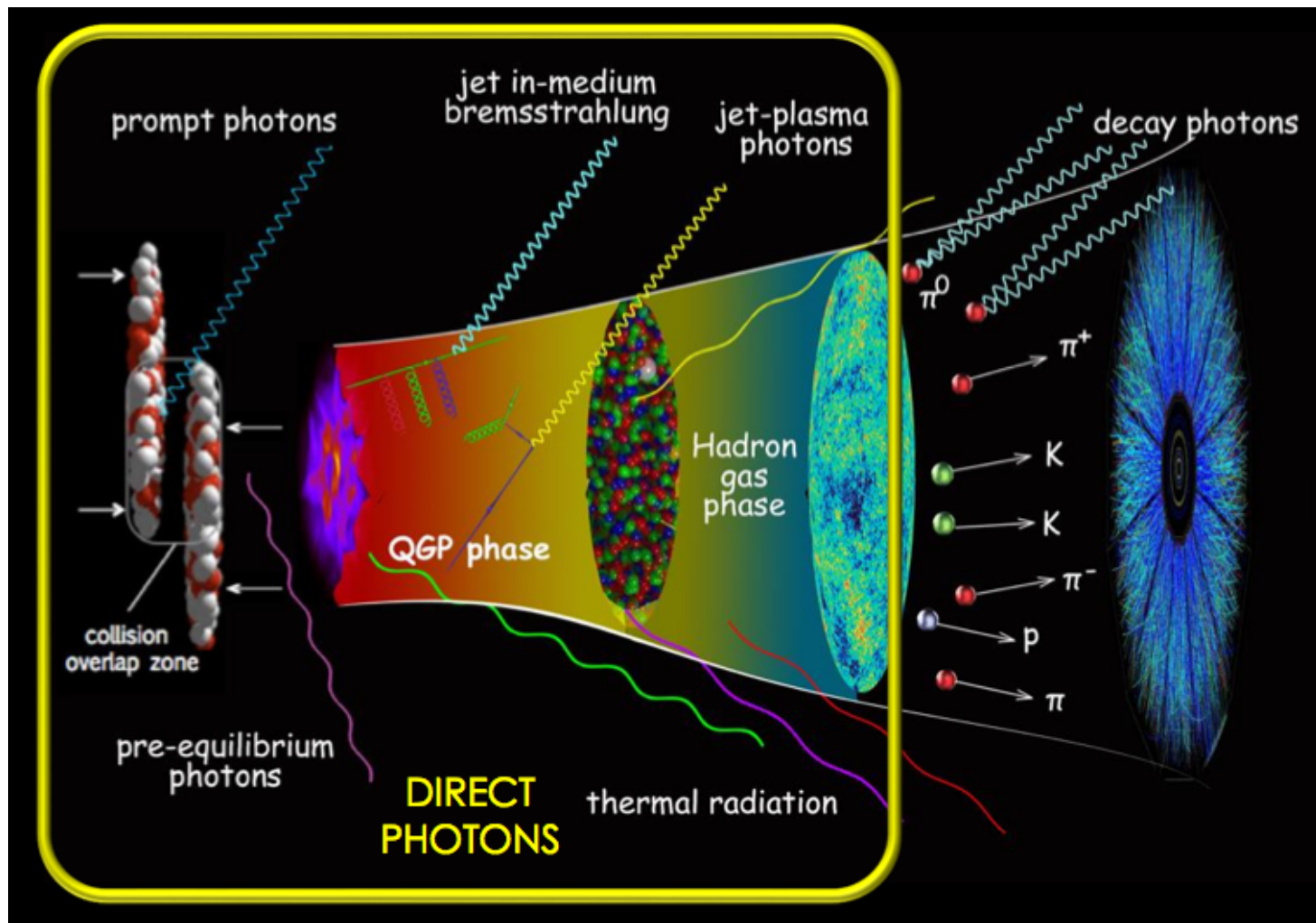
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S.I.Finazzo and R.Rougemont, PRD'16
I.Iatrakis, E.Kiritsis, C.Shen and D.L.Yang,
arXiv:1609.07208

DIRECT PHOTONS

- emerge directly from a particle collision
- represent less than 10% of all detected photons



[Source: C. Shen, talk at ECT*, Trento 12/2015]

Electric conductivity

Electric conductivity

$$\sigma \approx \left(\frac{2\pi\nu}{1+\nu} \right)^{3-2/\nu} \frac{T^{3-2/\nu}}{\left(1 - \left(\frac{\nu+1}{\nu} \right)^{\frac{2+3\nu}{\nu}} q^2 \left(\frac{1}{2\pi T} \right)^{\frac{2+4\nu}{\nu}} \right)^{3-2/\nu}}$$

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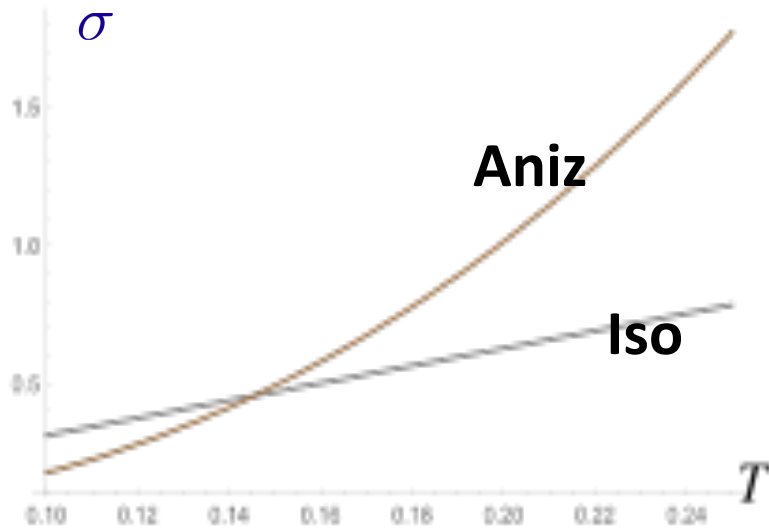
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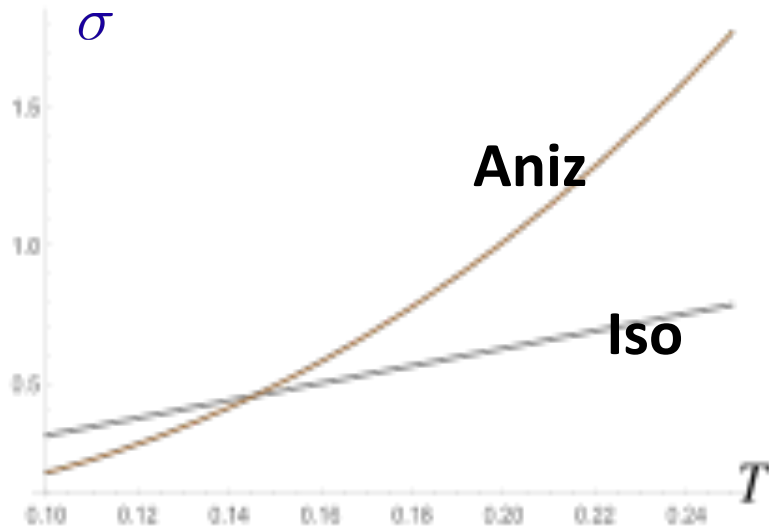
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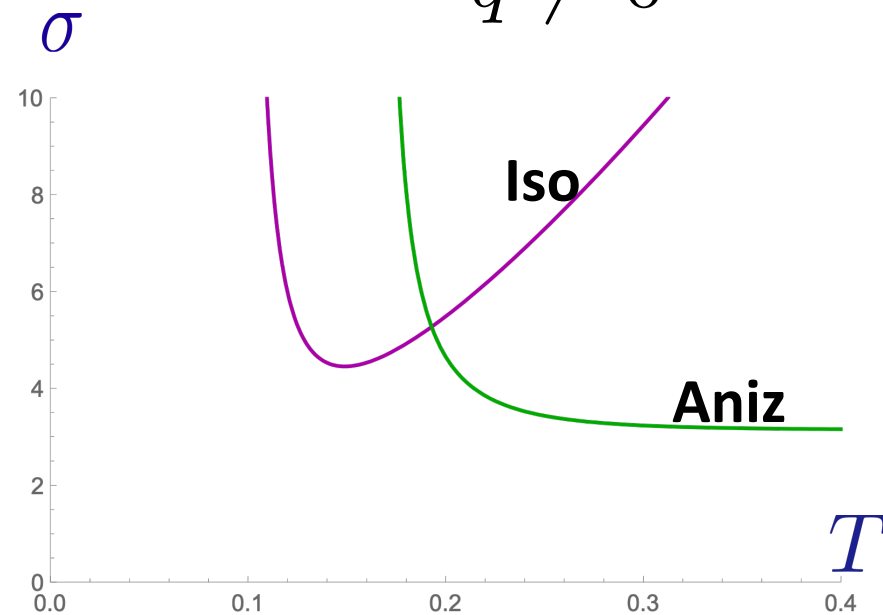
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$q \neq 0$

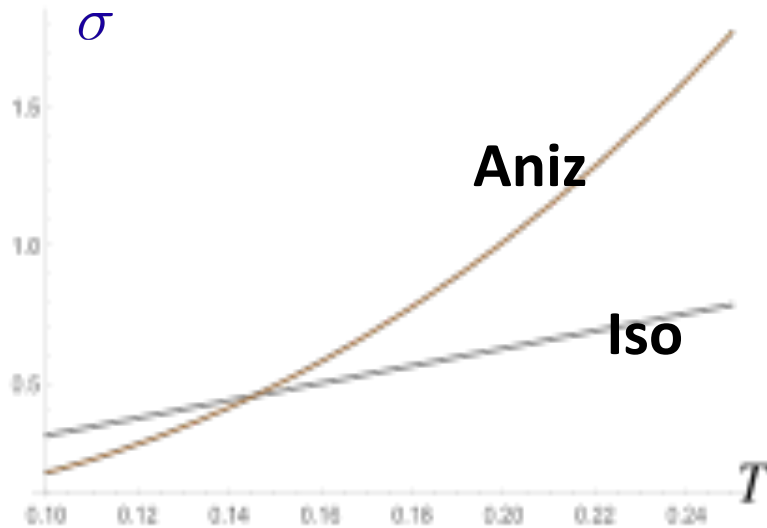


Increasing the anisotropy we increase the EC at $T > T_{cr}$, and vice versa for at $T < T_{cr}$

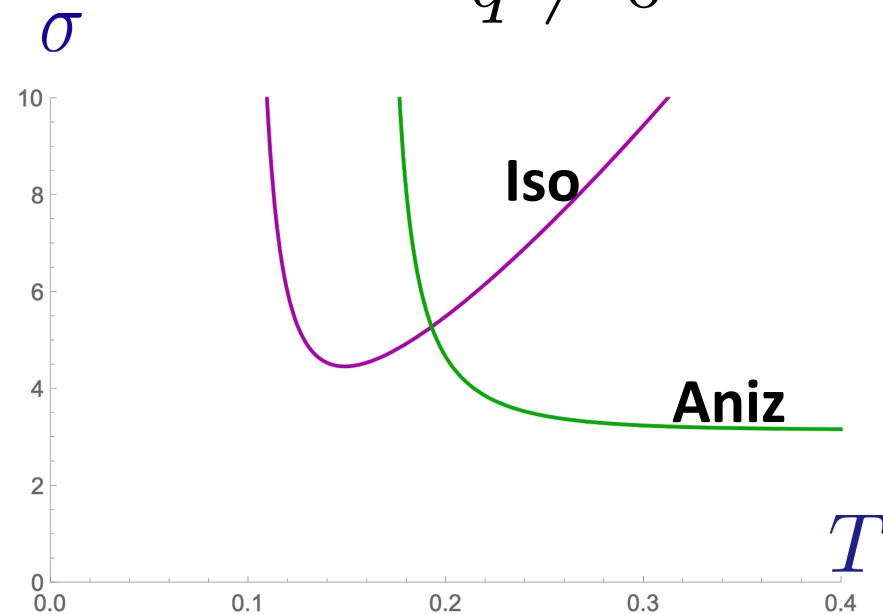
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We have considered the anizotropic model that **describes:**
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anizotropy in hadron spectrum (for a short time after collisions)

Anisotropy drastically change standard holographic calculations, in particular,

Wilson loops, and quark potential

Jet quenching

Drag forces

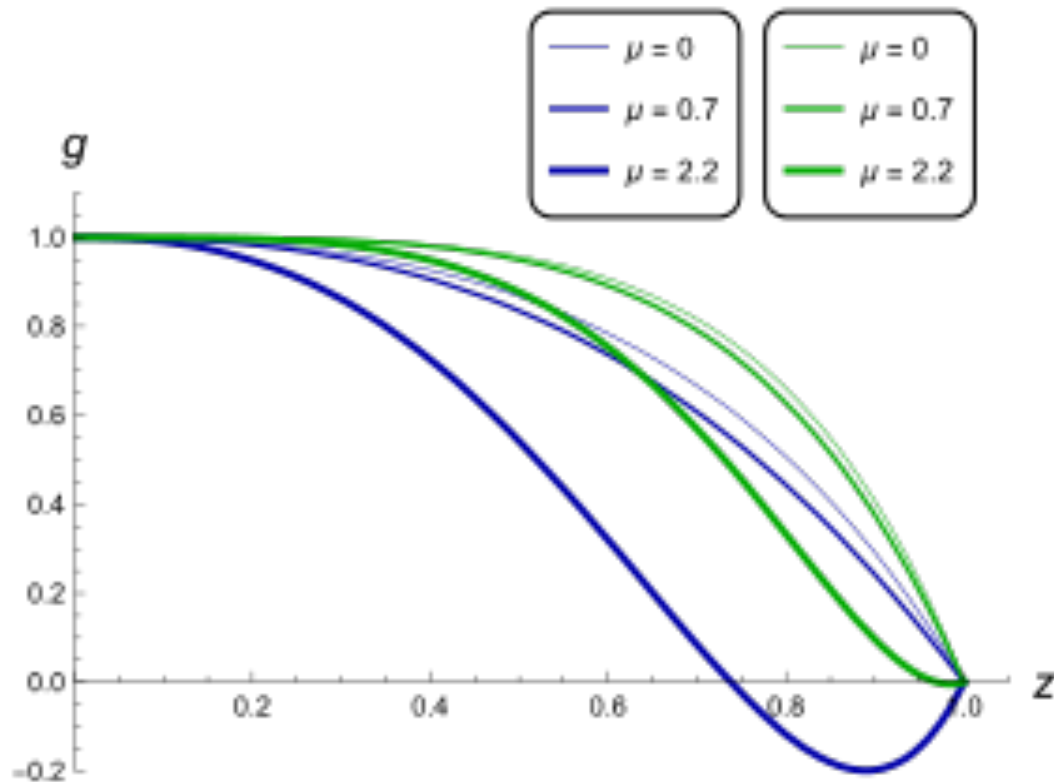
shear viscosity and therefore elliptic flows

susceptibility

thermalization time

BACKUP SLIDES

Details: Blackening function $g(z)$



$z_h = 1, \nu = 4.5, c = -1$ (blue lines)

$\nu = 1, c = -1$ (green lines)

Coupling function f_2

$$f_2(z) = \frac{\nu - 1}{q^2 \nu^2} z^{-\frac{4}{\nu}} e^{\frac{cz^2}{2}} \left[4(1 + \nu) - 3c\nu z^2 + 4 \frac{z^{2+\frac{2}{\nu}}}{z_h^{2+\frac{2}{\nu}}} \left\{ \frac{\nu e^{-\frac{3cz^2}{4}}}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)} - (1 + \nu) \frac{\mathfrak{G}\left(\frac{3}{4}cz^2\right)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)} \mathfrak{F} \right. \right. \\ \left. \left. + \frac{\mu^2 c \nu z_h^{2+\frac{2}{\nu}} e^{-cz^2 + \frac{cz_h^2}{2}}}{4 \left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \left(1 - e^{\frac{cz^2}{4}} \frac{\mathfrak{G}(cz_h^2)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)}\right) \right\} + 3c\nu \frac{z^{4+\frac{2}{\nu}}}{z_h^{2+\frac{2}{\nu}}} \frac{\mathfrak{G}\left(\frac{3}{4}cz^2\right)}{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)} \mathfrak{F} \right]$$

$$\mathfrak{F} = 1 - \frac{\mu^2 c z_h^{2+\frac{2}{\nu}} e^{\frac{cz_h^2}{2}}}{4 \left(1 - e^{\frac{cz_h^2}{4}}\right)^2} \left(\mathfrak{G}(cz_h^2) - \mathfrak{G}(cz^2) \frac{\mathfrak{G}\left(\frac{3}{4}cz_h^2\right)}{\mathfrak{G}\left(\frac{3}{4}cz^2\right)} \right).$$

Details: Scalar Field

$$\phi' = \frac{1}{\nu z} \sqrt{\frac{3}{2} \nu^2 c^2 z^4 - 9 \nu^2 c z^2 + 4\nu - 4}$$

$$\phi' = \frac{c}{z} \sqrt{\frac{3}{2} (\alpha^2 - z^2)(\beta^2 - z^2)},$$

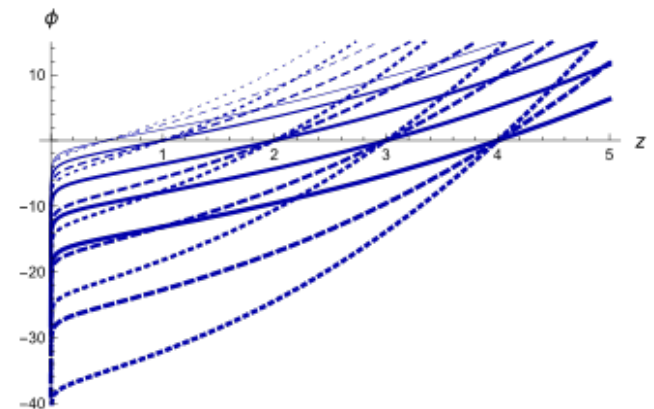
$$\alpha = \sqrt{\frac{3}{c} - \frac{1}{c} \sqrt{9 - \frac{8(\nu - 1)}{3\nu^2}}}, \quad \beta = \sqrt{\frac{3}{c} + \frac{1}{c} \sqrt{9 - \frac{8(\nu - 1)}{3\nu^2}}}.$$

$c > 0$ instability regions

$c = 0$

$$\phi = \frac{2\sqrt{\nu - 1}}{\nu} \ln \left(\frac{z}{z_h} \right)$$

$c < 0$



Details: **Blackening function** $g(z)$

$$b(z) = e^{\frac{cz^2}{2}}$$

$$\mathfrak{G}(x) = x^{-1-\frac{1}{\nu}} \gamma\left(1 + \frac{1}{\nu}, x\right)$$

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