Role of particle creation mechanism on the collapse of a massive star

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Acceleration of the Universe

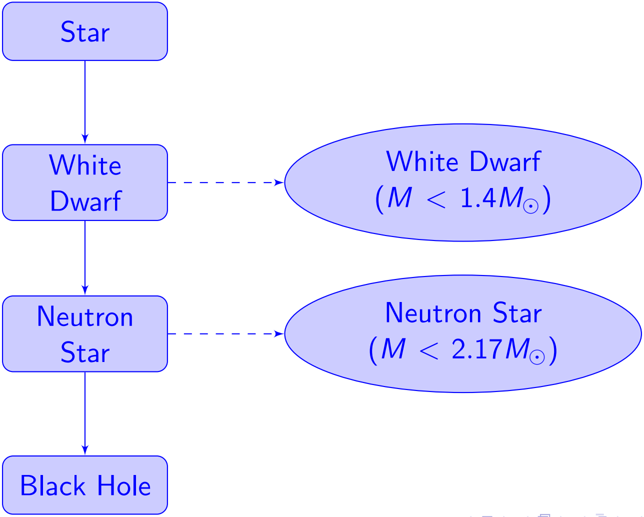
- Recent data [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] indicates that the expansion of our **Universe is accelerating**.
- To explain this phenomena either one has to modify matter or one has to modify geometry.
- To modify the matter term cosmologists introduced a component with negative pressure dubbed as **Dark Energy**.

Bulk viscosity in the accelerating Universe

- Bulk viscosity play an important role in the early stage of the Universe.
- Bulk viscosity can also describe present accelerating phase [10, 11].
- Origin of bulk viscosity: Interaction between different components or non-conservation of particle number.

Gravitational collapse

The gravitational collapse of a star follows as [12, 13, 14]



Laws and Hypothesis in gravitational collapse

- Singularity Theorem [15]
- Cosmic Censorship Conjecture [16]

Trapped Horizon and Singularity

- In trapped horizon both incoming and outgoing null geodesic converges [53, 54, 55].
- At singularity all the physical laws break down. Here pressure, density, curvature diverges.
- CCC [16] may be assumed to be related to the thermodynamic nature of the spacetime manifold near Naked Singularity (NS).

The basic set up

- The matter of the collapsing star is chosen in the form of perfect fluid with barotropic equation of state $p = (\gamma - 1)\rho$.
- The thermodynamic system is chosen as adiabatic. The effective bulk viscous pressure is determined by the particle creation rate [27, 28, 29, 30, 34] as

$$\Pi = -\frac{\Gamma}{3H}(p + \rho). \tag{1}$$

- The interior geometry is characterized by the flat Friedmann-Robertson-Walker (FRW) model

$$ds_-^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega_2^2). \tag{2}$$

- Gravitational collapse if $\dot{a} < 0$.

Trapped Surfaces and Apparent Horizon

- The apparent horizon is a trapped surface lying in a boundary of a particular surface S.
- For the present FRW model, the apparent horizon is characterized by [39, 40, 41]

$$R_{,i}R_{,j}g^{ij} \equiv (r\dot{a})^2 - 1 = 0. \tag{3}$$

- The comoving boundary surface of the star is spacelike: $r_{|\Sigma} = constant$, say r_Σ. Thus we have on Σ:

$$R_{,i}R_{,j}g^{ij} \equiv \{r_\Sigma \dot{a}(t)\}^2 - 1 < 0. \tag{4}$$

- Here r_Σ denotes the boundary of the collapsing star and we have on Σ:

$$ds_\Sigma^2 = d\tau^2 - R^2(\tau)d\Omega_2^2.$$

The Exterior Metric and the Mass of the Collapsing Cloud

- The metric outside the collapsing star in general can be written in the form [26, 42]

$$ds_+^2 = A^2(T,R)dT^2 - B^2(T,R)(dR^2 + R^2 d\Omega_2^2).$$

- The mass function due to Cahill and McVittie [43] is defined as

$$m(r,t) = \frac{R}{2}(1 + R_{,\alpha} R_{,\beta} g^{\alpha\beta}) = \frac{1}{2} R\dot{R}^2.$$

- Thus the total mass of the collapsing cloud is

$$m(\tau) = m(r_\Sigma, \tau) = \frac{1}{2} R(\tau)\dot{R}^2(\tau). \quad (5)$$

Basic Equations

- The basic Friedmann equations for the present model are

$$3H^2 = 8\pi G\rho \qquad \text{and} \qquad 2\dot{H} = -8\pi G(\rho + p + \Pi). \tag{6}$$

- Conservation equation

$$\dot{\rho} + 3H(\rho + p + \Pi) = 0. \tag{7}$$

Collapse Dynamics

- The collapse dynamics is characterized by the particle creation rate as

$$\frac{2\dot{H}}{3H^2} = -\gamma\left(1 - \frac{\Gamma}{3H}\right). \tag{8}$$

- In the present work, we shall choose Γ as [30]

$$\Gamma = \Gamma_3 + 3\Gamma_0 H + \frac{\Gamma_1}{H}.(\Gamma_0, \Gamma_1, \Gamma_3 \in \mathbb{R}, \Gamma \neq 0) \tag{9}$$

- The evolution of scale factor of the collapsing core

$$\frac{\ddot{a}}{a} + \left\{\frac{3\gamma}{2}\left(1 - \Gamma_0\right) - 1\right\}\frac{\dot{a}^2}{a^2} - \frac{\gamma\Gamma_3}{2}\frac{\dot{a}}{a} - \frac{\gamma\Gamma_1}{2} = 0. \tag{10}$$

The choices for Γ

We shall consider five choices:

- $\Gamma = \Gamma_3 + 3\Gamma_0 H + \frac{\Gamma_1}{H}$.
- $\Gamma = \Gamma_3 + 3H + \frac{\Gamma_1}{H}$.
- $\Gamma = 3\Gamma_0 H$.
- $\Gamma = 3H + \Gamma_1/H$.
- $\Gamma = \Gamma_3 + 3\Gamma_0 H$

Choice I: $\Gamma = \Gamma_3 + 3\Gamma_0 H + \frac{\Gamma_1}{H}$

- We consider the evolution equation (10)

$$\frac{\ddot{a}}{a} + \left\{\frac{3\gamma}{2}\left(1 - \Gamma_0\right) - 1\right\}\frac{\dot{a}^2}{a^2} - \frac{\gamma\Gamma_3}{2}\frac{\dot{a}}{a} - \frac{\gamma\Gamma_1}{2} = 0. \qquad (11)$$

- The solutions for rate of contraction and scale factor

$$H = [-H_2^{-1} + \mu \tanh T]^{-1} \qquad (12)$$

$$\left(\frac{a}{a_0}\right)^{\mu\alpha_1} = e^{lT}\left[H_2\left\{\frac{\Gamma_3}{2\Gamma_1}\cosh T - \mu \sinh T\right\}\right]^m. \qquad (13)$$

- $\mu^2 = \frac{\{12\Gamma_1(1-\Gamma_0)+\Gamma_3^2\}}{4\Gamma_1^2}$, $\alpha_1 = \frac{\gamma\Gamma_1}{2}$, $m = \frac{\mu}{[\mu^2-(\frac{\Gamma_3}{2\Gamma_1})^2]}$,

  $l = \frac{H_2^{-1}}{[\mu^2-H_2^{-2}]}$, $T = \mu\alpha_1(t - t_0)$, $H_2 = (\frac{\Gamma_3}{2\Gamma_1})^{-1}$.
- $t_c = t_0 + (\mu\alpha_1)^{-1}\left[\tanh^{-1}\left(1/H_2\mu\right)\right]$.

- Time of formation of apparent horizon

$$R_0 H_2 e^{\left(\frac{l}{\mu\alpha_1}\right)T_{aH}}\left(\cosh T_{aH}\right)^{n+1}\left[1-\frac{\tanh T_{aH}}{\tanh T_c}\right]^n = 1.$$

- $T_{aH} = \mu\alpha_1(t_{aH} - t_0)$, $R_0 = a_0 r$ and $n = \frac{m}{\mu\alpha_1} - 1$.
- $t_c > t_{aH}$ for any real value of n (except n to be a positive integer).
- $t_c < t_{aH}$ or $t_c > t_{aH}$, if n is an even integer.

Choice II: $\Gamma = \Gamma_3 + 3H + \frac{\Gamma_1}{H}$

- The evolution equation (10) simplifies to

$$\dot{H} = \frac{\gamma}{2}(\Gamma_3 H + \Gamma_1). \tag{14}$$

- The solutions for scale factor and rate of contraction

$$H = -\delta + (H_0 + \delta)e^{-\frac{\gamma\alpha}{2}(t-t_0)}$$
$$a = a_0 e^{-\delta(t-t_0)} \exp\left[-\frac{2(H_0+\delta)}{\gamma\alpha}\left\{e^{-\frac{\gamma\alpha}{2}(t-t_0)} - 1\right\}\right]. \tag{15}$$

- $\alpha = -\Gamma_3$, $\mu = -\Gamma_1$ and $\delta = \frac{\Gamma_1}{\Gamma_3}$.
- $t_c = \infty$.

- Time of formation of apparent horizon

$$R_0 e^{-\delta \widetilde{T}_{aH}} \left[\delta - (H_0 + \delta) e^{-\frac{\gamma\alpha}{2} \widetilde{T}_{aH}}\right] \exp\left[-\frac{2(H_0+\delta)}{\gamma\alpha}\left\{e^{-\frac{\gamma\alpha}{2}\widetilde{T}_{aH}} - 1\right\}\right] = 1 \quad (16)$$

- $\widetilde{T}_{aH} = t_{aH} - t_0$.
- t_{aH} always has a finite solution.

- The measure of acceleration is given by

$$\frac{\ddot{a}}{a} = \Big[-\delta + (H_0 + \delta) e^{-\frac{\gamma\alpha}{2}(t - t_0)} - \frac{\gamma\alpha}{4} \Big]^2 - \Big(\frac{\gamma^2\alpha^2}{16} - \frac{\gamma\mu^2}{2} \Big). \quad (17)$$

- Accelerating if $t > t_0 + \frac{2}{\gamma\alpha} ln\Big(\frac{\delta + H_0}{\delta + \frac{\gamma\alpha}{4} - \sqrt{\frac{\gamma^2\alpha^2}{16} - \frac{\gamma\mu}{2}}}\Big)$ or $t < t_0 + \frac{2}{\gamma\alpha} ln\Big(\frac{\delta + H_0}{\delta + \frac{\gamma\alpha}{4} + \sqrt{\frac{\gamma^2\alpha^2}{16} - \frac{\gamma\mu}{2}}}\Big)$
- Decelerating if $t_0 + \frac{2}{\gamma\alpha} ln\Big(\frac{\delta + H_0}{\delta + \frac{\gamma\alpha}{4} + \sqrt{\frac{\gamma^2\alpha^2}{16} - \frac{\gamma\mu}{2}}}\Big) < t < t_0 + \frac{2}{\gamma\alpha} ln\Big(\frac{\delta + H_0}{\delta + \frac{\gamma\alpha}{4} - \sqrt{\frac{\gamma^2\alpha^2}{16} - \frac{\gamma\mu}{2}}}\Big)$.

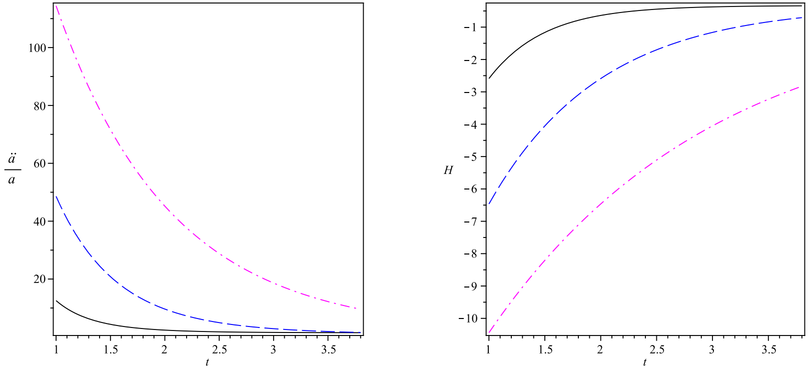


Figure: The figure on the left side represents accelerating collapsing process against time t given by (17) and the figure on the right side denotes evolution of the rate of contraction (H), which is given by (15) against t, respectively for $\Gamma_0 = 1$. In both the figures, the curves in the solid line represent $\frac{\ddot{a}}{a}$ and H, respectively for $\gamma = \frac{4}{3}$. The curves in the dashed line represent $\frac{\ddot{a}}{a}$ and H, respectively for $\gamma = \frac{2}{3}$ and the curves in the dash-dotted line represent $\frac{\ddot{a}}{a}$ and H, respectively for $\gamma = \frac{1}{3}$. $\alpha = 3$,

Choice III: $\Gamma = 3\Gamma_0 H$

- The evolution equation simplifies to ($\Gamma_0 \neq 1$)

$$\frac{\ddot{a}}{a} + \left\{ \frac{3\gamma}{2}(1-\Gamma_0) - 1 \right\} H^2 = 0, \qquad (18)$$

- The solutions for scale factor and rate of contraction

$$H = \frac{H_0}{\left[1 + \frac{3\gamma H_0}{2}(1-\Gamma_0)(t-t_0)\right]}. \qquad (19)$$

$$a = a_0 \left[1 + \frac{3\gamma H_0}{2}(1-\Gamma_0)(t-t_0)\right]^{\frac{2}{3\gamma(1-\Gamma_0)}}.$$

- $t_c = t_0 - \frac{2}{3\gamma H_0(1-\Gamma_0)}$.

- Time of formation of apparent horizon

$$t_{aH} = t_0 + \frac{2}{3\gamma H_0(1-\Gamma_0)}\left[-1+\left(-\frac{1}{R_0 H_0}\right)^{\frac{1}{l}}\right], \quad l = \frac{2}{3\gamma(1-\Gamma_0)} - 1.$$

- $t_{aH} - t_c = \frac{1}{H_0}\left(-\frac{1}{R_0 H_0}\right)^{\frac{1}{l}}$
- $t_{aH} < t_c$

- The measure of acceleration is given by

$$\frac{\ddot{a}}{a} = \frac{\{1 - \frac{3\gamma}{2}(1 - \Gamma_0)\} H_0^2}{\left[1 + \frac{3\gamma H_0}{2}(1 - \Gamma_0)(t - t_0)\right]^2} \tag{20}$$

- Accelerating if $\frac{3\gamma}{2}(1 - \Gamma_0) < 1$
- Decelerating if $\frac{3\gamma}{2}(1 - \Gamma_0) > 1$.

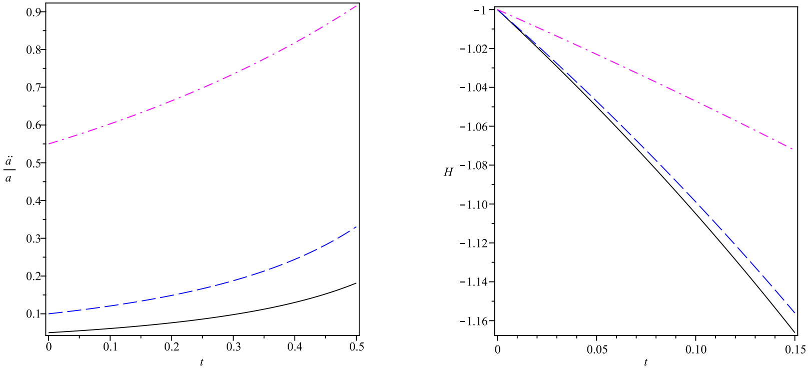


Figure: The figure on the left side depicts accelerating collapsing process (given by the first case of (20)) and the figure on the right side depicts evolution of the rate of contraction (H) given by (19), respectively against time t for $\Gamma_1 = 0$. The curves in the solid line represent $\frac{\ddot{a}}{a}$ and H, respectively for $\Gamma_0 = -0.9$, the dashed lines represent for $\Gamma_0 = -0.8$ and the dash-dotted lines represent for $\Gamma_0 = 0.1$, , respectively. In all the figures we have considered $\gamma = \frac{1}{3}$

Choice IV: $\Gamma = 3H + \Gamma_1/H$

- The evolution equation simplifies to

$$\dot{H} = \frac{\gamma\Gamma_1}{2}. \tag{21}$$

- The solutions for scale factor and rate of contraction

$$\begin{aligned} H &= H_0 + \tfrac{\gamma\Gamma_1}{2}(t - t_0). \\ a &= a_0 \exp\left[H_0(t - t_0) + \tfrac{\gamma\Gamma_1}{4}(t - t_0)^2\right]. \end{aligned} \tag{22}$$

- We choose $\Gamma_1 < 0$.
- $t_c = \infty$.

- Time of formation of apparent horizon

$$R_0\left[H_0+\frac{\gamma\Gamma_1}{2}T_{aH}\right]\exp\left[H_0T_{aH}+\frac{\gamma\Gamma_1}{4}T_{aH}^2\right]=-1,$$

- t_{aH} always has a finite solution.

- The measure of acceleration is given by

$$\frac{\ddot{a}}{a} = \left\{ H_0 + \frac{\gamma\Gamma_1}{2}(t - t_0) \right\}^2 + \frac{\gamma\Gamma_1}{2}. \qquad (23)$$

- Decelerating if $-\sqrt{-\frac{\gamma\Gamma_1}{2}} - H_0 < \frac{\gamma\Gamma}{2}(t - t_0) < \sqrt{-\frac{\gamma\Gamma_1}{2}} - H_0.$

- Otherwise accelerating.

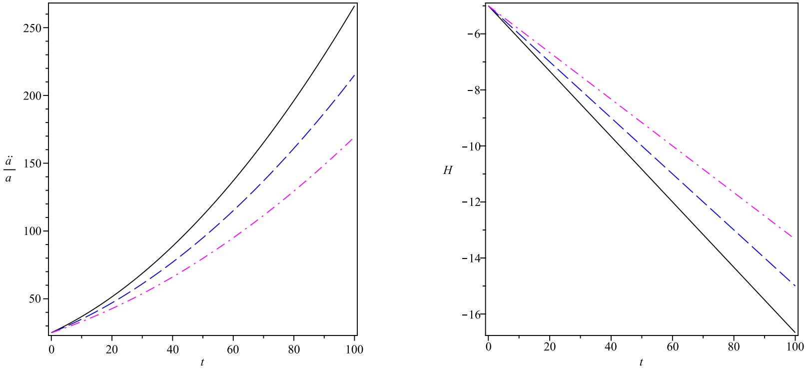


Figure: The figure on the left side shows accelerating collapsing process given by (23) and the figure on the right side shows evolution of rate of contraction given by (22), respectively against time t. In both the figures the curves in the solid line represent $\frac{\ddot{a}}{a}$ for $\Gamma_1 = -0.7$, the dashed lines represent for $\Gamma_1 = -0.6$, and the dash-dotted lines represent for $\Gamma_1 = -0.5$, respectively. In all the cases here we have considered $\gamma = \frac{1}{3}$, $t_0 = 0$.

Choice V: $\Gamma = \Gamma_3 + 3\Gamma_0 H$

- The evolution equation simplifies to

$$\frac{\ddot{a}}{a} + \left\{\frac{3\gamma}{2}(1-\Gamma_0) - 1\right\}\frac{\dot{a}^2}{a^2} - \frac{\gamma\Gamma_3}{2}\frac{\dot{a}}{a} = 0. \qquad (24)$$

- The solutions for scale factor and rate of contraction

$$a = a_0 \exp\left[\frac{2H_0}{\gamma\Gamma_3}\left\{e^{\frac{\gamma\Gamma_3}{2}(t-t_0)} - 1\right\}\right],$$
$$H = H_0 \exp\left[\frac{\gamma\Gamma_3}{2}(t-t_0)\right], \qquad (25)$$

- We choose $\Gamma_3 > 0, \Gamma_0 = 1$.
- $t_c = \infty$.

- Time of formation of apparent horizon

$$R_0 H_0 e^{\frac{\gamma\Gamma_3}{2}(t_{aH}-t_0)} \exp\left[\frac{2H_0}{\gamma\Gamma_3}\left\{e^{\frac{\gamma\Gamma_3}{2}(t_{aH}-t_0)} - 1\right\}\right] = -1.$$

- t_{aH} always has a finite solution.

- The measure of acceleration is given by

$$\frac{\ddot{a}}{a} = H_0 e^{\frac{\gamma\Gamma 3}{2}(t-t_0)}\left[H_0 e^{\frac{\gamma\Gamma 3}{2}(t-t_0)} + \frac{\gamma\Gamma_3}{2}\right]. \tag{26}$$

- Accelerating if $t > t_0 + \frac{2}{\gamma\Gamma_3} ln(-\frac{\gamma\Gamma_3}{2} H_0)$
- Otherwise decelerating.

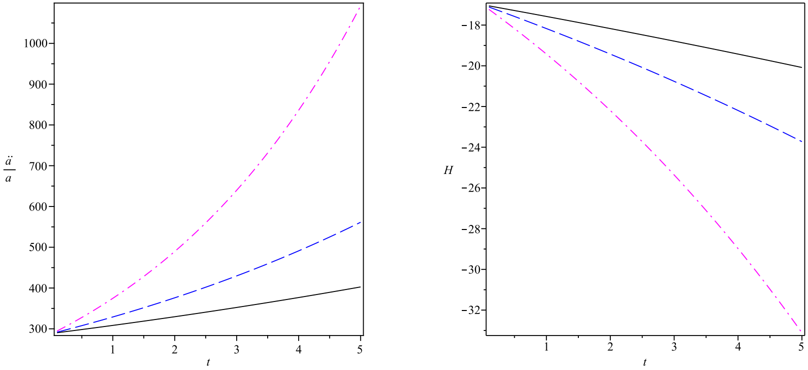


Figure: The figure on the left side shows $\frac{\ddot{a}}{a}$ (given by (26)) vs. time t and the figure on the right side shows evolution of the rate of contraction given by (25) for $\Gamma = 3H + \Gamma_3, \ \Gamma_3 > 0$. In both the figures, the curves in the solid line represent $\frac{\ddot{a}}{a}$ for $\gamma = \frac{1}{3}$, the dashed lines represent for $\gamma = \frac{2}{3}$ and the dash-dotted lines represent for $\gamma = \frac{4}{3}$, respectively. In all the cases here we have considered $\Gamma_3 = 0.2, \ t_0 = 0$.

Conclusion

- We have measured the acceleration and rate of contraction during collapse.
- We have definite conclusion about the end state as Black Hole.
- We may also have end state as Black Hole or Naked Singularity but Black hole is more favoured.

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Thank You