Toward traversable stable semiclassical asymptotically flat wormholes

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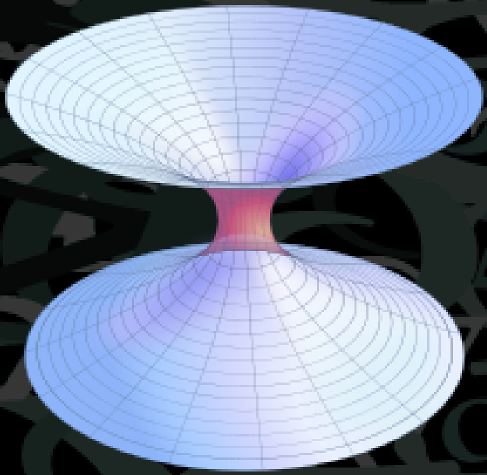
Wormholes

- In principle easy to build – but we need to violate energy conditions
- Spherical wormhole (WH) in (3+1)d:

$$ds^2 \sim -F dt^2 + G dr^2 + r^2 d\Omega^2$$

John Wheeler

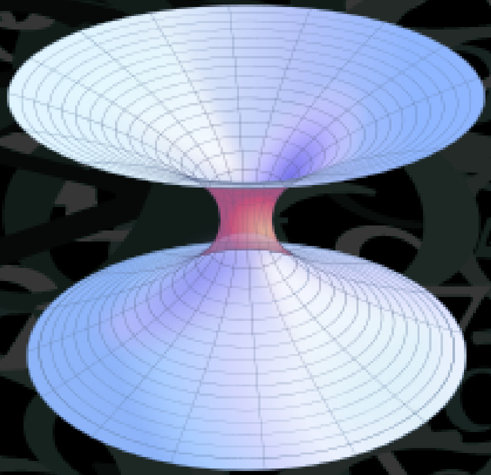


No horizon, no infinite redshift

Kip Thorne

Wormholes and black hole information paradox

- Idea: wormhole instead of a black hole ⇒ no horizon, no Hawking radiation, no possible information loss
- Challenge 1: WH should look like a BH from far away, otherwise impossible to square with abundant empirical and astrophysical support for BH (or BH-like?) objects



$$ds^2 \sim -F dt^2 + G dr^2 + r^2 d\Omega^2$$

No horizon, no infinite redshift

F, G close to BH solution

Wormholes and black hole information paradox

- Idea: wormhole instead of a black hole $\Rightarrow$ no horizon, no Hawking radiation, no possible information loss
- Challenge 1: WH should look like a BH from far away, otherwise impossible to square with abundant empirical and astrophysical support for BH (or BH-like?) objects
- Answered in Maldacena, Milekhin & Popov 2018 [1807.04726] in flat space, by Maldacena et al, Horowitz et al, Freivogel et al in AdS space
- Challenge 2: WH should be reasonably stable, possible to build from ordinary matter, and energetically and thermodynamically preferred to BH

Outline

- Building stable asymptotically flat blackened wormholes in a natural way:

  – from semiclassical matter

  – from stringy matter

- Toward wormhole thermodynamics – Bekenstein and Kolmogorov-Sinai entropy [1904.06295]

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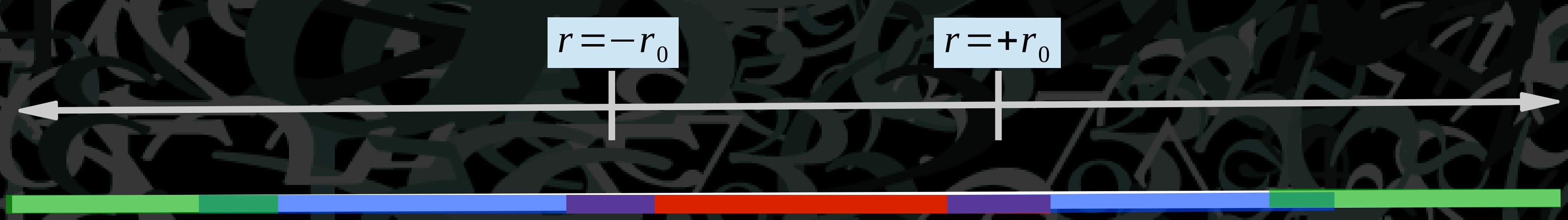
The goal

- Semiclassical – $T_{\mu\nu}$ first order in $\hbar$, gravity action zeroth order in $\hbar$
- BH-like: throat, intermediate region (like neutral or charged BH), outer region (becomes flat)



Strategy: match the expansions in the overlapping regions

The goal

- Semiclassical – $T_{\mu\nu}$ first order in $\hbar$, gravity action zeroth order in $\hbar$
- BH-like:

$|r|$

throat region:

$|r-r_0| \ll r_0$

$$ds_2^2 \sim -dt^2(1+r^2) + \frac{dr^2}{1+r^2}$$

AdS-like

intermediate region:

$|r-r_0| \sim r_0$

$$ds^2 \sim -f dt^2 + dr^2/f + r^2 d\Omega^2$$

BH-like: $f \sim 4\pi T(r-r_0) + O(\hbar)$

outer region:

$|r-r_0| \gg r_0$

$$ds^2 \sim -dt^2 + dx^2$$

flat

Scalar field WH solution

- We know classical minimally coupled scalar obeys NEC
- What about a perturbation coming from infinity?
- One-loop correction of the stress-energy: $T_{\mu\nu} = T^{(0)}_{\mu\nu} + \epsilon \langle T^{(1)}_{\mu\nu} \rangle$

$$\langle T^{(1)}_{\mu\nu}(x) \rangle = \lim_{x' \to x} \partial_{x'_\mu} \partial_{x_\mu} \delta G(x, x') - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \left(\partial_{x'_\rho} \partial_{x_\sigma} \delta G(x, x') - m^2 \delta G(x, x') \right)$$

- Metric perturbation from Schwarzschild BH:

$$g_{tt} = f(r) + \epsilon \gamma_1(r), \quad g_{rr} = \frac{1}{f(r) + \epsilon \gamma_1(r)}, \quad g_{\varphi\varphi} = g_{\theta\theta} \sin^2\theta = r^2 (1 + \epsilon \gamma_2(r)) \sin^2\theta$$

- Perturbation eqs:

$$r \gamma_2' - \gamma_2 - 8\pi (1 + r^2) \langle T^{(1)}_{rr} \rangle = 0$$

$$\gamma_1'' + (1 + r^2) \gamma_2'' + 2r \gamma_2' + 4\gamma_2 - 16\pi \langle T^{(1)}_{\theta\theta} \rangle = 0$$

Traversable WH solution

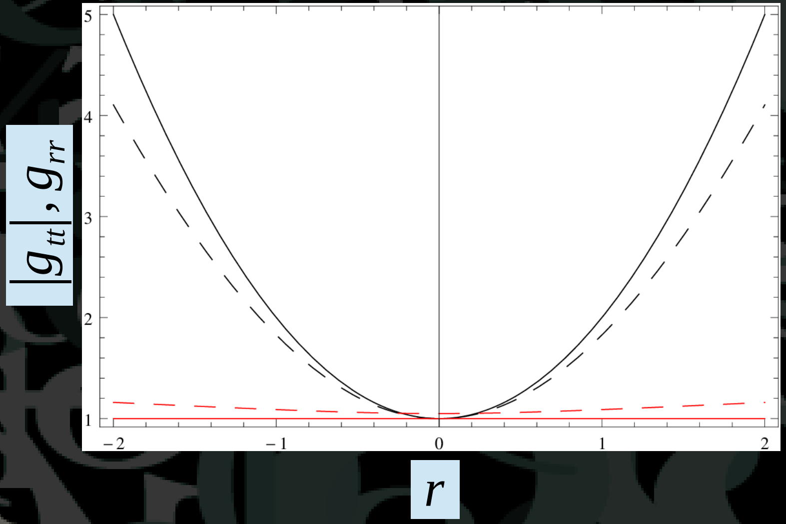
- Solution: $\gamma_2(r) = 8\pi r \int dr' \frac{1+r'^2}{r'^2} \langle T_{rr}(r') \rangle$
- Analytical expression for $G(x,y)$ and Bessel function identities yield the condition for traversable WH from matching at $r - r_0 \sim r_0$:

$$\gamma_2(r \rightarrow r_0) = C \times (r - r_0)^2 + \ldots, \quad C > 0$$

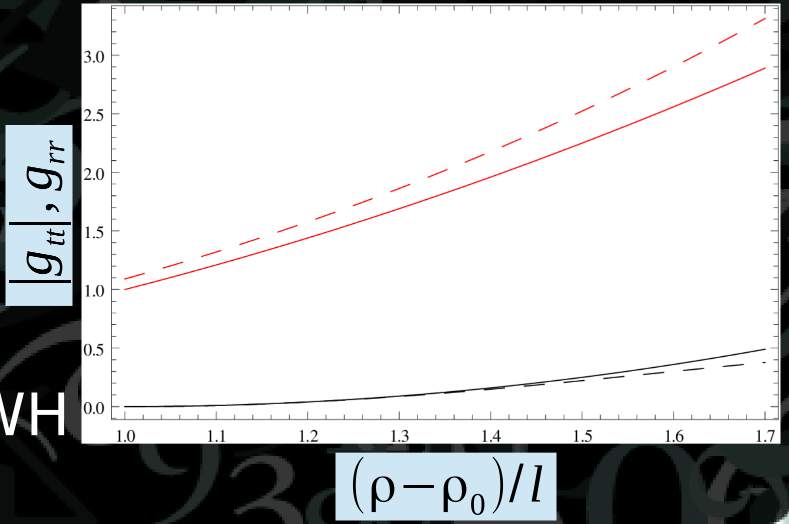
- Impossible without nonlocal coupling!

Fermionic perturbation

- Dirac fermion ψ : $T_{\mu\nu} = T^{(0)}_{\mu\nu} + \epsilon \langle T^{(1)}_{\mu\nu} \rangle$ – loop correction only
- Solve the Dirac equation by matching the throat region and the intermediate region [1302.5149]
- In the throat: $\psi(t, r, \theta, \varphi) = \sum_{\omega} \sum_{j} \chi_{\omega l}(r) e^{il\varphi}$
- Outcome: $\langle T^{(1)}_{rr} \rangle = \frac{\omega}{(1+r^2)^2} (\sin 2\alpha - \cos 2\alpha), \quad \tan 2\alpha = -m/\omega$



Full – BH
Dashed – WH



Fermionic perturbation

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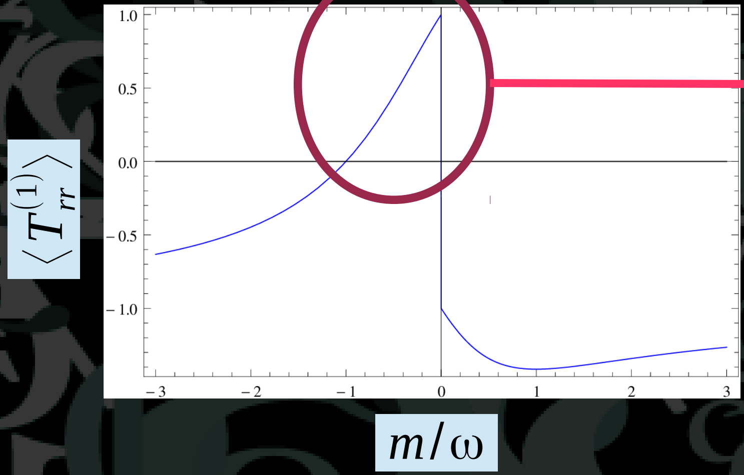

not allowed: $\omega < 0$ and $|\omega| > m$

Fermionic perturbation

0

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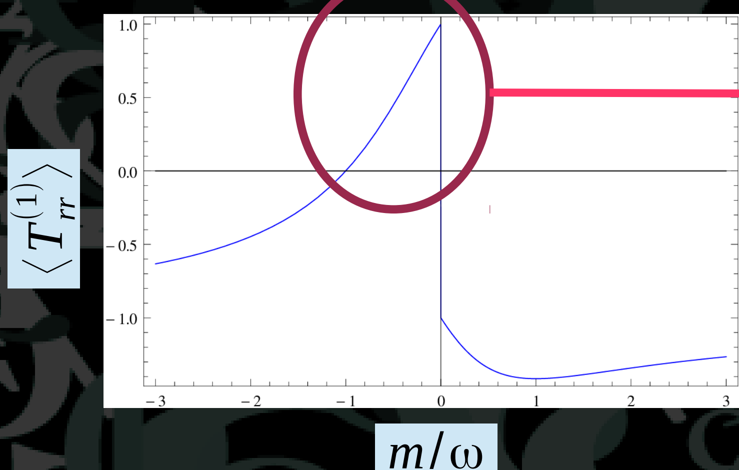

not allowed: $\omega<0$ and $|\omega|>m$

Need to introduce a chemical potential to renormalize the Dirac sea of negative energies

Fermionic perturbation

- Dirac fermion ψ : $T_{\mu\nu} = T^{(0)}_{\mu\nu} + \epsilon \langle T^{(1)}_{\mu\nu} \rangle$ – loop correction only
  0
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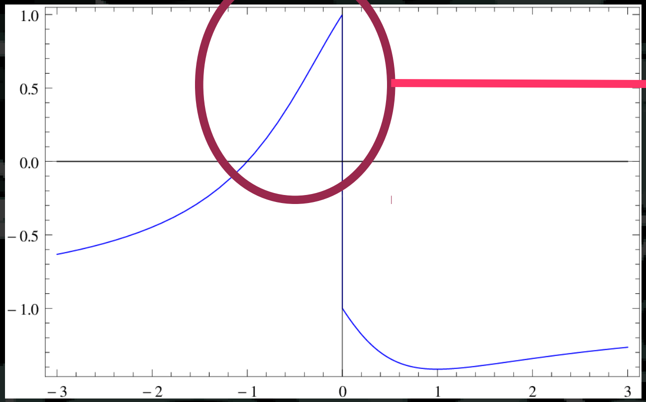

$\langle T^{(1)}_{rr} \rangle$

m/ω

not allowed: $\omega < 0$ and $|\omega| > m$

For $m = 0$ no issue, this is the case from Maldacena, Milekhin and Popov 2018

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Pair-creation

- The simplest process: breaking an open string

$$\text{open string} \begin{cases} \nearrow \text{Pair of BHs} \\ \searrow \text{WH – traversable?} \end{cases}$$

- Semiclassically: Nambu-Goto action S_{NG} vs. two BH action S_{2BH} vs. WH action S_{WH}

$$S_{NG} = \frac{1}{\pi\alpha'} \int d\tau \int d\sigma \sqrt{-\det\gamma}$$

$$S_{2BH} = -2m \int d\tau \sqrt{-g^{(0)}_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu}}$$

BH pair ~ pair of particles

$$S_{WH} = -m \int d\tau \sqrt{-g^{(1)}_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu}}$$

Single WH

Pair-creation

- The simplest process: breaking an open string

open string → Pair of BHs

open string → WH – traversable?

- Semiclassically: Nambu-Goto action S_{NG} vs. two BH action S_{2BH} vs. WH action S_{WH}

- Important difference from semiclassical matter: now the spacetime topology can change dynamically
- Not eternal WH but dynamically generated WH
- Makes sense to ask questions about thermodynamics

Mechanical equilibrium

- The BH horizons coincide at creation (scenario long known, eg Thorne, Letelier) so there is a WH
- But: BHs accelerate away from each other ⇒ WH more and more fragile
- Seek an equilibrium solution @ fixed throat length:

$$L=\sqrt{F\left(r(t)\right)-G\left(r(t)\right)\dot{r}^2}$$

- Effective potential: $V(r)=\frac{F}{G}\left[\left(m/S_{NG}\right)^2-1\right]$
- Equilibrium condition: $V'(r_0)=0$
- Ansatz for the metric:

$$g_{tt}=f(r)+\epsilon\gamma_1(r)P_l(\cos\theta),\quad g_{rr}=\frac{1}{f(r)}+\epsilon\gamma_2(r)P_l(\cos\theta)$$

WH solution

- Einstein equations horrible but analytically solvable through hypergeometric functions
- WH throat develops if $1/\alpha' \ll 1/r_0^2$

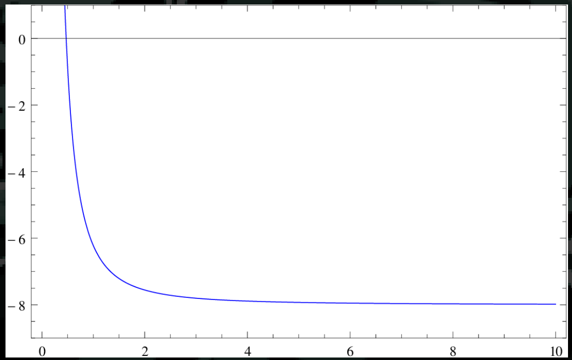
  Always satisfied for a semiclassical string

- Acceleration small if

  $|V''(r)| \lesssim 1/r_0^2$

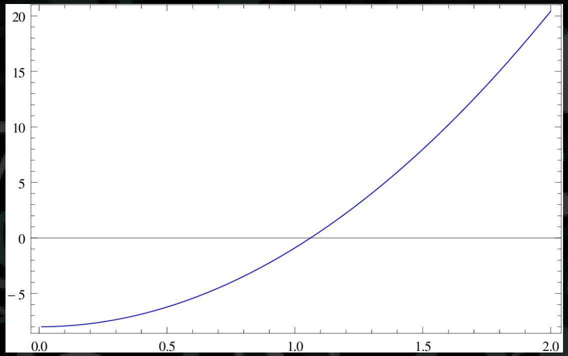
- Again always true as long as $S_{NG} \ll 1/l_P$

$\langle T_{rr} \rangle$



$1/\alpha'$

$\delta V''(r)$



S_{NG}

Thermodynamic equilibrium

- BH entropy from Hawking-Bekenstein: $S_{2BH} = 2 \times A/4 = \pi r_0^2/2$
- WH entropy: $S_{WH} = E_{\text{gap}}/T = (r_0/d)/a$
- Magnetic WH (Maldacena et al 2018): $a = 1/2\pi d$

$S_{WH} = 2\pi r_0 \rightarrow S_{2BH}/S_{WH} \sim r_0 \gg 1$

- For a stringy WH acceleration suppressed by r_0^2:

$a = V''(r_0) r_0^2 \sim d/r_0^2 \longrightarrow S_{WH} = E_{\text{gap}}/T = r_0^3/d^2 \gg S_{BH} = \pi r_0^2/2$

- Now WH both mechanically and thermodynamically stable but the price to pay is high: going for strings

Entropy production

What is the physical interpetation?

Making a WH from BH produces entropy:

$$S_{WH} = E_{\mathrm{gap}} / T = r_0^3 / d^2 \gg S_{BH} = \pi r_0^2 / 2$$

$$S_{WH} - S_{BH} > 0$$

Entropy counts degrees of freedom.

Where do the extra degrees of freedom come from?

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KS entropy

- Natural measure of complexity, far ahead of its time (late 1950s)

$$h = \sum_{\lambda_i > 0} \lambda_i$$

KS entropy is the sum of positive Lyapunov exponents

А. Н. Колмогоров, Новый метрический инвариант транзитивных динамических систем и автоморфизмов пространств Лебега, Докл. АН СССР, 119:5 (1958)

Я.Г. Синай, О понятии энтропии динамической системы, Докл. АН СССР, 124(4), 768-771 (1959).

Ring string in WH background

- Polyakov action:

$$S=\frac{1}{2\pi\alpha'}\int d\tau\, d\sigma\left(\eta_{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu+\epsilon_{ab}B_{\mu\nu}\partial_a X^\mu \partial_b X^\mu\right)$$

- Same gauge, same Virasoro constraints:
- Ansatz: ring configuration $T(\tau), R(\tau), \Phi_1(\tau);\;\; \Phi_2(\sigma)=n\sigma$
- Integral of motion:

$E=\dot{T}f(R)=\text{const.}$

moves freely in radius and latitude

wrapped along longitude

- Effective Hamiltonian:

$$H_{\text{eff}}=\frac{f(R)}{2}P_R^2+\frac{1}{2R^2}P_\Phi^2+\frac{E^2}{2f(R)}+n^2R^2\,\text{sink}^2\Phi$$

Lyapunov exponents and KS entropy

- Variational equations yield Lyapunov exponents:

$$\ddot{\delta} R - fR\left(\dot{\Phi}^2 - \operatorname{sink}^2\Phi\right) - \frac{\dot{f}}{2f}\left(\dot{R}^2 - E^2\right) = 0$$

$$\ddot{\delta}\Phi + 2\frac{\dot{R}}{R}\dot{\Phi} + \frac{n^2}{2}\operatorname{sink}\left(2\Phi\right) = 0$$

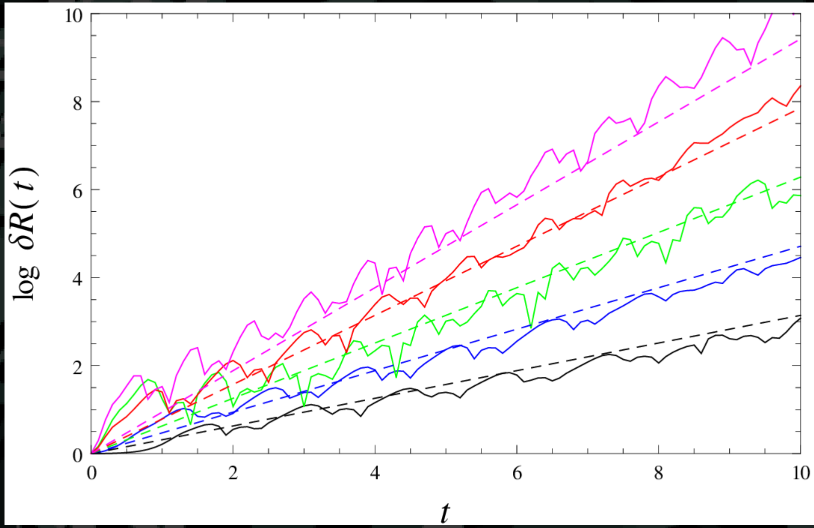
- Outcome at order ϵ:

$$\lambda_R = \frac{r_0 - d}{r_0^2 d V''} - \pi r_0^2/2, \quad \lambda_\Phi = \frac{1}{r_0^2 V''}$$

$$h = \lambda_R + \lambda_\Phi = \frac{r_0}{da} - \frac{\pi r_0^2}{2} = S_{WH} - S_{BH}$$

Lyapunov exponents and KS entropy

Numerical check for the total divergence rate (h)



Full lines: numerical solution of the variational equations

Dashed lines: theoretical prediction $h = S_{WH} - S_{BH}$

Lyapunov exponents and KS entropy

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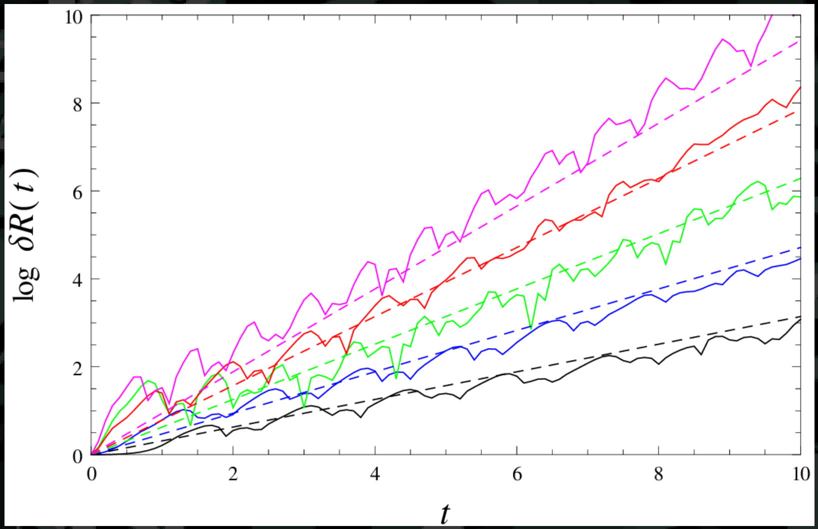
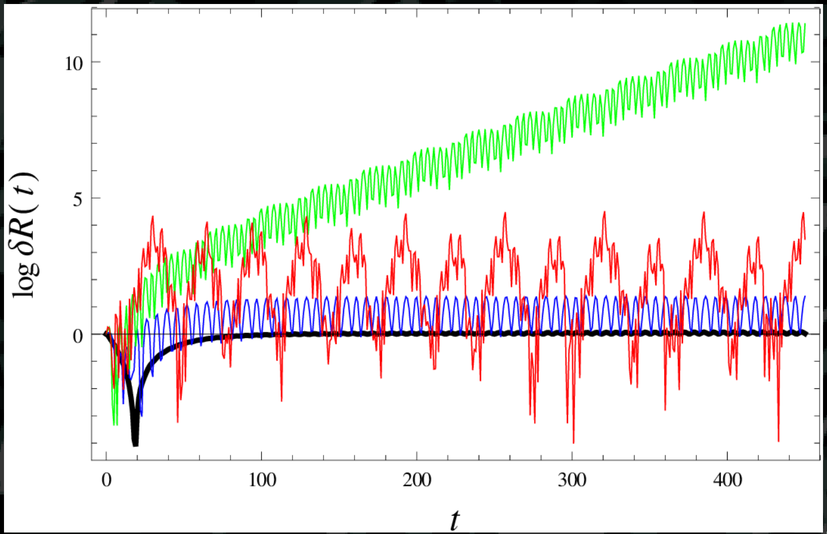


Full lines: numerical solution of the variational equations
Dashed lines: theoretical prediction $h = S_{WH} - S_{BH}$

Black, blue: KS entropy for unstable WH ($S_{WH} - S_{BH} < 0$): $h = 0$

Red, green: KS entropy for the marginal case $S_{WH} = S_{BH}$



Open string in WH background

- Polyakov action:

0

$$S=\frac{1}{\pi\alpha'}\int d\tau\, d\sigma\left(\eta_{ab}G_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}+\epsilon_{ab}B_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\mu}\right)$$

- Gauge $h_{ab}=\eta_{ab}$ $\Rightarrow$ Virasoro constraints:

$$\eta_{ab}G_{\mu\nu}\partial_a X^{\mu}\partial_b X^{\nu}=0,\quad \epsilon_{ab}\partial_a X^{\mu}\partial_b X^{\nu}=0$$

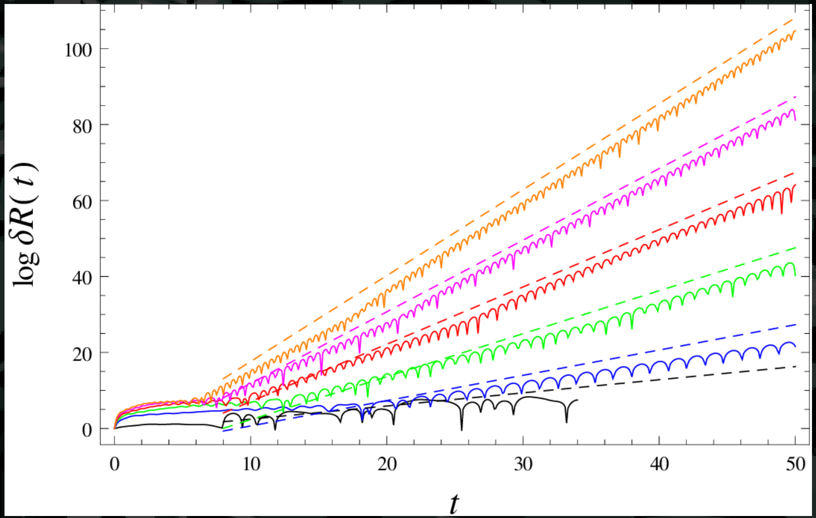
- Ansatz: string hangs from infinity to infinity through WH

$$T(\tau),R(\tau),\Phi_1=\text{const.};\Phi_2=\text{const.}$$

- Fluctuations $\epsilon\Phi_1(\tau,\sigma),\ \Phi_2(\tau,\sigma)$
- No integrals of motion survive

Lyapunov exponents and KS entropy

Numerical check for the total divergence rate (h)



Full lines: numerical solution of the variational equations
Dashed lines: theoretical prediction $h = S_{WH} - S_{BH}$

- Analytically at leading order likewise:

$$\lambda_R = \frac{2-\pi}{r_0 d V''} - \frac{\pi r_0^2}{4}, \quad \lambda_\Phi = \frac{\pi}{r_0 d V''} - \frac{\pi r_0^2}{4}, \quad h = S_{WH} - S_{BH}$$

Instead of a conclusion

Thd entropy of BHs & WHs

KS entropy of WH hair

Microstates of hair & BHs

Transport in Anosov systems

- Almost all reviews & textbooks: no connection between KS entropy and thermodymic entropy
- Exception: Lorentz gas and similar systems (Gaspard et al, van Beijeren et al 1980s)
- Applicable here? Relation to BH microstates?