Causality in nonlocal gravity

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Introduction

- Renormalizability and unitarity are requiremets that can hardly be reconciled within a consistent theory of quantum gravity.
- Einstein-Hilbert gravity is non-renormalizable, but, if we include infinitely many counterterms, it is pertubatively unitary.
- Renormalizable higher-derivative theories of gravity (e.g. Stelle's quadratic theory) can be attained, but are generically expected to be non-unitary.
- Recently, [Camanho, Edelstein, Maldacena, Zhiboedov,'16] it has been argued that higher-derivative corrections to the 3-graviton coupling in a weakly coupled theory of gravity are constrained by causality.

Weakly nonlocal gravity

We consider the model

$$S_{\rm g} = \frac{2}{\kappa_D^2} \int d^D x \sqrt{-g}\, [R + G_{\mu\nu}\gamma(\Box) R^{\mu\nu} + V(\mathcal{R})] \,, \tag{1}$$

where ($\sigma \equiv \ell_\Lambda^2$)

$$\gamma(\Box) = \frac{e^{H(\sigma\Box)} - 1}{\Box} \,. \tag{2}$$

- $\exp H(z)$ is asymptotically polynomial

$$\exp H(z) \to |z|^{\gamma + \mathrm{N} + 1} \quad \text{for} \quad |z| \to +\infty, \quad \gamma \geqslant \frac{D}{2} \,, \tag{3}$$

  with $2\mathrm{N} + 4 = D_{even}$ or $2\mathrm{N} + 4 = D_{odd} + 1$, to guarantee the locality of counterterms.
- $V(\mathcal{R}) \sim \mathcal{O}(\mathcal{R}^3)$, but quadratic in the Ricci tensor, is a local potential containing at most $2\gamma + 2N + 4$ derivatives.

Super-renormalizability and finiteness

- By standard power-counting

$$\delta^D(K)\,\Lambda^{2\gamma(L-1)}\int(d^Dp)^L\left(\frac{1}{p^{2\gamma+D}}\right)^I\left(p^{2\gamma+D}\right)^V$$

  we get the degree of divergence $\omega(G)\equiv D_{even}-2\gamma(L-1)$ and $\omega(G)\equiv D_{odd}-(2\gamma+1)(L-1)$.
- if $\gamma>(D_{odd}-1)/2$, no divergences!
- if $\gamma>D_{even}/2$, only 1-loop divergences !
- Some terms in $V(\mathcal{R})$ can be used as "killers" of the 1-loop divergences. For example, in $D=4$, the two terms

$$s_1R^2\Box^{\gamma-2}(R^2),\quad s_2R_{\mu\nu}R^{\mu\nu}\Box^{\gamma-2}(R_{\rho\sigma}R^{\rho\sigma}),\tag{4}$$

  modify the beta-functions for R and $R^2_{\mu\nu}$ by a contribution linear in s_1 and s_2, making it possible to have them vanishing. The killer terms should be in general at least quadratic in the Ricci tensor.

Perturbative unitarity

- In the harmonic gauge $(\partial_\mu h^{\mu\nu} = 0)$

$$\mathcal{O}^{-1} \approx \frac{P^{(2)}}{k^2 e^{H(k^2/\Lambda^2)}} - \frac{P^{(0)}}{(D-2)\, k^2 e^{H(k^2/\Lambda^2)}} \,. \tag{5}$$

No ghosts appear if $H(z)$ are entire functions with no poles.

- The usual analytic continuation from Euclidean to Minkowski cannot be performed due to thebehavior at infinity of $\exp H$, but [Modesto,Briscese,2018], [Pius,Sen,2016] still the ordinary Cutkosky rules can be derived and it is possible to prove at all perturbative levels the unitarity relation

$$T_{ab} - T^*_{ba} = i \sum_c T^*_{cb} T_{ca} \tag{6}$$

Quadratic gravity

The most general gravity action quadratic in the curvatures is

$$S_g = -2\kappa_D^{-2} \int d^D x \sqrt{-g} \Big(R + \gamma_0' R^2 + \gamma_2' R_{\mu\nu}^2 + \gamma_4 \mathrm{GB} \Big) \,, \tag{7}$$

Major advantages

- GB gives no contribution to the propagator for any D (neither to the vertices in $D = 4$, being topological)
- expanding around a flat background ($R^{(0)} = R^{(0)}_{\mu\nu} = R^{(0)}_{\mu\nu\rho\sigma} = 0$), vertices are greatly simplified by the relationships $\sqrt{-g}^{\,(1)} = R^{(1)} = R^{(1)}_{\mu\nu} = 0$ valid for on-shell legs.
- Three level amplitudes with all external legs on graviton shell are calculable by standard techniques

Scattering amplitudes for Stelle's theory

Born approximation four graviton scattering amplitudes in the center-of-mass reference frame, $s = 4E^2$, $t = -2E^2(1 - \cos\theta)$ and $u = -2E^2(1 + \cos\theta)$

$$\mathcal{A}(++,++) = \mathcal{A}_s(++,++) + \mathcal{A}_t(++,++) + \mathcal{A}_u(++,++) + \mathcal{A}_{\text{contact}}(++,++)$$
$$= -2i\left(-\frac{2}{\kappa_4^2}\right)E^2\frac{1}{\sin^2\theta}\,,$$

- The amplitude doesn't have the expected UV behavior $\sim E^4$ and is the same as the one determined in Einstein gravity by dimensional analysis and symmetry arguments. This is the result of non-trivial cancellation between the massive poles in the propagator and the three-graviton vertices and between the contact and exchange diagrams.
- Our result is consistent with the fact that in the absence of the Einstein term we are left with scale invariant terms whose contribution to amplitudes for dimensionless particles is vanishing.

For $D > 4$ the Gauss-Bonnet term contributes the vertices

$$\mathcal{A}^{D=5}(++,++)$$
$$= -i\frac{2}{\kappa_5^2}\left\{\frac{16E^6\gamma_4^2\left[1+8E^2\left(3(\gamma_0-\gamma_4)+(\gamma_2+4\gamma_4)\right)\right]}{(1-4E^2(\gamma_2+4\gamma_4))\left[3+4E^2\left(16(\gamma_0-\gamma_4)+5(\gamma_2+4\gamma_4)\right)\right]} - 2E^2\frac{1}{\sin^2\theta}\right\}$$
$$\mathcal{A}^{D=6}(++,++)$$
$$= -i\frac{2}{\kappa_6^2}\left\{\frac{8E^6\gamma_4^2\left[1+8E^2\left(3(\gamma_0-\gamma_4)+(\gamma_2+4\gamma_4)\right)\right]}{(1-4E^2(\gamma_2+4\gamma_4))\left[1+2E^2\left(10(\gamma_0-\gamma_4)+3(\gamma_2+4\gamma_4)\right)\right]} - 2E^2\frac{1}{\sin^2\theta}\right\}$$

- In $D > 4$ the expected linear term in γ_4 is absent due to a non trivial cancellation between contact and exchange diagrams.
- In $D > 4$ the dependence on γ_0' and γ_2' is due to the fact that in exchange diagrams the massive poles cannot cancel with the three-graviton vertices of GB. This is associated to the dependence on arbitrary power of E in the IR.

Scattering amplitudes for weakly nonlocal gravity

If $\gamma_0' = \gamma_0'(\Box)$, $\gamma_2' = \gamma_2'(\Box)$ and $\gamma_4 = 0$,

$$\mathcal{A}_s(++,++) = -2\kappa_4^{-2}\left(-\frac{9}{8}\frac{t(s+t)}{s} + \frac{9}{32}\gamma_2(s)\left(s^2 + (s+2t)^2\right) + \frac{9}{8}s^2\gamma_0(s)\right), \tag{8}$$

$$\mathcal{A}_t(++,++) = -2\kappa_4^{-2}\left(-\frac{1}{8}\frac{\left(s^3 - 5s^2t - st^2 + t^3\right)(s+t)^2}{s^3 t}\right.$$
$$\left. + \frac{1}{16}\gamma_2(t)\frac{\left(2s^4 - 10s^3t - s^2t^2 + 4st^3 + t^4\right)(s+t)^2}{s^4} + \frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4}\right), \tag{9}$$

$$\mathcal{A}_u(++,++) = -2\kappa_4^{-2}\left(-\frac{1}{8}\frac{\left(s^3 - 5s^2u - su^2 + u^3\right)(s+u)^2}{s^3 u}\right.$$
$$\left. + \frac{1}{16}\gamma_2(u)\frac{\left(2s^4 - 10s^3u - s^2u^2 + 4su^3 + u^4\right)(s+u)^2}{s^4} + \frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\right), \tag{10}$$

$$\mathcal{A}_{\text{contact}}(++,++) = -2\kappa_4^{-2}\left(-\frac{1}{4}\frac{s^4 + s^3t - 2st^3 - t^4}{s^3} - \frac{9}{32}\gamma_2(s)\left(s^2 + (s+2t)^2\right) - \frac{9}{8}s^2\gamma_0(s)\right.$$
$$- \frac{1}{16}\gamma_2(t)\frac{\left(2s^4 - 10s^3t - s^2t^2 + 4st^3 + t^4\right)(s+t)^2}{s^4} - \frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4}$$
$$\left. - \frac{1}{16}\gamma_2(u)\frac{\left(2s^4 - 10s^3u - s^2u^2 + 4su^3 + u^4\right)(s+u)^2}{s^4} - \frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\right). \tag{11}$$

The cancellation of poles occurs separately in each channel

$$\mathcal{A}(++,++) = \mathcal{A}(++,++)_{\text{EH}}\,. \tag{12}$$

A field redefinition theorem

Given two actions $S'(g)$ and $S(g)$ such that

$$S'(g) = S(g) + E_i(g)F_{ij}(g)E_j(g)\,, \tag{13}$$

where F_{ij} can contain derivatives and $E_i = \delta S/\delta g_i$, there exist a field redefinition

$$g'_i = g_i + \Delta_{ij}E_j \quad \Delta_{ij} = \Delta_{j\,i}, \tag{14}$$

such that, perturbatively in F and to all orders in powers of F, we have the equivalence

$$S'(g) = S(g')\,. \tag{15}$$

- The theorem states the equivalence of the two theories only perturbatively in F. In particular the two theories are clearly different if $S'(g)$ has additional poles wrt $S(g')$.
- The theorem in particular applies to tree-level amplitudes whenever the external legs are on the mass-shell shared by the two theories.

Implications for higher derivative theories

- Any higher derivative gravity theory which can be recast in the form

$$S'(g) = S_{\rm EH}(g) + R_{\mu\nu}(g) F^{\mu\nu,\rho\sigma}(g) R_{\rho\sigma}(g)\,. \tag{16}$$

  with $F^{\mu\nu,\rho\sigma} = g^{\mu\nu} g^{\rho\sigma} \gamma_0(\Box) + g^{\mu\rho} g^{\nu\sigma} \gamma_2(\Box) + \tilde{\mathbf{V}}(\mathbf{R}, \mathbf{Ric}, \mathbf{Riem}, \nabla)^{\mu\nu\rho\sigma}$, shares the same n-graviton on-shell tree-level amplitude as $S_{\rm EH}(g)$.
- If we neglect finite contributions to the quantum effective action, this result can be applied to all finite weakly nonlocal theories with $\gamma_4(\Box) = 0$ and killers of the kind $R^2 \Box^{\gamma-2}(R^2)$ and $R_{\mu\nu} R^{\mu\nu} \Box^{\gamma-2}(R_{\rho\sigma} R^{\rho\sigma})$. It also applies to 1-loop super-renormalizable theories in $D = 4$, while in general terms giving non vanishing contribution will be generated in renormalizable and super-renormalizable theories for $D \geq 6$.
- More in general the theorem applies whenever the finite contributions to the quantum effective action can be cast in such a way as to be at least quadratic in the scalar curvature and Ricci tensors.
  $\Longrightarrow$ Higher derivatives terms contain crucial physical information about the UV behavior, but, at least in some cases, look quite elusive observationally.

Causality: Shapiro's time delay

- One possible definition of causality [Gao,Wald,'00] is that we cannot send signals faster than what is allowed by the asymptotic causal structure of the spacetime.
- We want to probe the scale $\Lambda_{PL} \ll b \ll \ell_\Lambda$.
- In the limit $t/s << 1$ (but large s) we consider the Eikonal approximation

$$iA_{\text{eik}} = 2s \int d^{D-2}\vec{b}\, e^{-i\vec{q}\cdot\vec{b}} \left[e^{i\delta(b,s)} - 1\right] , \tag{17}$$

where the phase is given by

$$\delta(b,s) = \frac{1}{2s} \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}} A_{\text{tree}}(s, -\vec{q}^{\,2}) \,. \tag{18}$$

The result is independent on the particular theory (higher derivative or weakly non-local)
Shapiro's time delay is:

$$\Delta t = 2\partial_E \delta(E,b) \,. \tag{19}$$

where E is the energy of the probe-particle.

Causality Violation in Gauss-Bonnet gravity

- For the theory

$$\mathcal{L} = \frac{2}{\kappa_D^2}\left[R + \lambda_{\rm GB}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\right]. \quad (20)$$

we have $A_t = A_{t{\rm EH}} + A_{t{\rm GB}}$, where

$$\begin{aligned} A_{t{\rm EH}} &\approx -\frac{8\pi G s^2}{t}(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4) \\ A_{t{\rm GB}} &\approx \frac{\kappa_D^2\lambda_{\rm GB}s^2}{t}\left(k_2^\mu k_4^\nu \epsilon_{2\nu}^\rho \epsilon_{4\rho\mu}\epsilon_1\cdot\epsilon_3 + k_1^\mu k_3^\nu \epsilon_{1\nu}^\rho \epsilon_{3\rho\mu}\epsilon_2\cdot\epsilon_4\right). \end{aligned}$$

- One can find ($\vec{n} \equiv \vec{b}/b$)

$$\begin{aligned} \Delta t_{\rm g-GB} &= \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{16EG}{b^{D-4}}(\epsilon_1\cdot\epsilon_1)(\epsilon_2\cdot\epsilon_2) \\ &\left[1 + \frac{4\lambda_{\rm GB}(D-2)(D-4)}{b^2}\left(\frac{(n\cdot\epsilon_1)^2}{\epsilon_1\cdot\epsilon_1} + \frac{(n\cdot\epsilon_2)^2}{\epsilon_2\cdot\epsilon_2} - \frac{2}{D-2}\right)\right] \end{aligned} \quad (21)$$

which can be negative for $b^2 < \lambda_{\rm GB}$.

Causality in pure nonlocal gravity

- The leading four-graviton amplitude in the Regge limit is:

$$A_{NL}(++,++) = A_t(++,++) = -\frac{8\pi G s^2}{t}. \tag{22}$$

- The phase and Shapiro's time delay are respectively:

$$\delta_{\rm g}(b,s) = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{Gs}{b^{D-4}} (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4)\,, \tag{23}$$

$$\Delta t_{\rm g} = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}} \frac{16EG}{b^{D-4}} (\epsilon_1 \cdot \epsilon_3)(\epsilon_2 \cdot \epsilon_4)\,. \tag{24}$$

no time advance $\implies$ no causality violation

Scalar field in nonlocal gravity

- In the limit $t \ll s$ the leading contribution comes from the amplitude in the t-channel, namely

$$A_t(s,t) \approx -8\pi G \frac{s^2}{t} e^{-H(t)} \,. \tag{25}$$

- In $D=5$

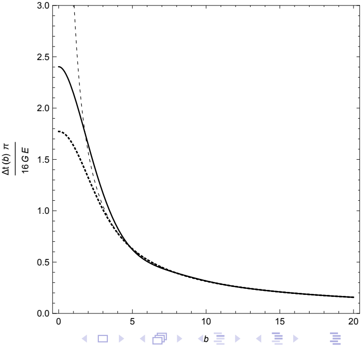
$$\delta(b,s) = \frac{1}{2s}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{i\vec{q}\cdot\vec{b}} A_t(s,-\vec{q}^{\,2}) = \frac{2Gs}{\pi}\int dq \frac{\sin(bq)}{bq}\, e^{-H(-q^2)} \,, \tag{26}$$

- For the form factor $e^{-\sigma\Box}$

$$\delta(b,s)_{\rm SFT} = Gs\,\frac{\mathrm{Erf}(b/2\ell_\Lambda)}{b} \,,$$

- It reduces to the one in Einstein's theory for $b \gg \ell_\Lambda$, namely

$$\delta(b,s)_{\rm SFT} \quad \to \quad \delta_{\rm EH}(b,s) = \frac{Gs}{b} .$$



General causal nonlocal theories

- *Any nonlocal theory that is tree-level equivalent to a causal local one is causal too. Given a causal (possibly local) theory, the field redefinition theorem provides an algorithm for constructing a full class of higher derivative (even non-local) causal theories.*
- A remarkable example

$$\mathcal{L} = \frac{2}{\kappa_D^2}\left[R + \left(G_{\mu\nu} - \frac{\kappa_D^2}{2}(T^A_{\mu\nu} + T^\phi_{\rho\sigma})\right) F_{\rm g}^{\mu\nu,\rho\sigma} \left(G_{\rho\sigma} - \frac{\kappa_D^2}{2}(T^A_{\rho\sigma} + T^\phi_{\rho\sigma})\right)\right]$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \nabla_\mu F^{\mu\nu}\, F^{\rm A}\, \nabla_\rho F^\rho{}_\nu + \frac{1}{2}\phi(\Box - m^2)\phi + \phi(\Box - m^2)\, F^\phi\, (\Box - m^2)\phi\,,$$

where

$$F_{\rm g}^{\mu\nu,\rho\sigma} \equiv \left(g^{\mu\rho}g^{\nu\sigma} - \frac{1}{2}g^{\mu\nu}g^{\rho\sigma}\right)\left(\frac{e^{H_{\rm g}(\Box)} - 1}{\Box}\right),$$

$$F^{\rm A} \equiv \frac{1}{2}\left(\frac{e^{H_A(\Box)} - 1}{\Box}\right) \qquad T^A_{\mu\nu} \equiv F_{\mu\sigma}F^\sigma_\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,,$$

$$F^\phi \equiv \frac{1}{2}\left(\frac{e^{H_\phi(\Box - m^2)} - 1}{\Box - m^2}\right), \quad T^\phi_{\mu\nu} \equiv \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial_\lambda\phi\partial^\lambda\phi + m^2\phi^2)\,.$$

Conclusions

- In recent years, nonlocal theories have proved to be much more treatable than expected. In particular, such issues like quantum renormalizability and perturbative unitarity seems to be not unreconcilable.
- In particular, we can choose which kind of higher derivative terms can show up using for example causality as a guide principle.
- Similar method can be applied to $N = 1$ nonlocal supergravity.
- Future directions of research can be $N > 1$ nonlocal supergravites, role of conformal symmetry in achieving finiteness, singularity-free solutions, etc.

THANKS!