Noncommutative $OSp(4|2)$ SUGRA

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Deformation quantization

1. *Deformation quantization* (phase space quantum mechanics). Classical system $(\mathcal{M}, \omega, H)$ is deformed by imposing noncommutative (NC) geometry on its phase space ; $\star$-product deformation of commutative algebra $C^\infty(\mathcal{M})$.
2. NC Field Theory - field theory on NC-deformed space-time. Introduce an abstract algebra of coordinates

$$[\hat{x}^\mu, \hat{x}^\nu] = iC^{\mu\nu}(\hat{x}) \ .$$

Canonical (or θ-constant) deformation,

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \sim \Lambda_{NC}^2 \ ,$$

with constant deformation parameters $\theta^{\mu\nu} = -\theta^{\nu\mu}$.

For *canonical noncommutativity*, we use the Moyal $\star$-product,

$$(\hat{f} \star \hat{g})(x) = e^{\frac{i}{2}\, \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu}} f(x)g(y)|_{y\to x} \ .$$

The leading term is the commutative point-wise multiplication, and the higher order terms represent (non-classical) NC corrections.

NC gauge field theory

Let $\{T_A\}$ satisfy some Lie algebra relations $[T_A, T_B] = if_{AB}{}^C T_C$. Closure rule holds

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{-i[\epsilon_1,\epsilon_2]} \ .$$

If NC gauge parameter $\hat{\Lambda}$ is supposed to be Lie algebra-valued, $\hat{\Lambda}(x) = \hat{\Lambda}^A(x) T_A$, then, for some generic NC field $\hat{\Psi}$ from the fund. rep.

$$\begin{aligned}[\delta_1^\star \overset{\star}{,} \delta_2^\star]\hat{\Psi} &= (\hat{\Lambda}_1 \star \hat{\Lambda}_2 - \hat{\Lambda}_2 \star \hat{\Lambda}_1) \star \hat{\Psi} \\ &= \frac{1}{2}\Big([\hat{\Lambda}_1^A \overset{\star}{,} \hat{\Lambda}_2^B]\{T_A, T_B\} + \{\hat{\Lambda}_1^A \overset{\star}{,} \hat{\Lambda}_2^B\}[T_A, T_B]\Big) \star \hat{\Psi} = i\hat{\Lambda}_3 \star \hat{\Psi} = \delta_3^\star \hat{\Psi} \ .\end{aligned}$$

NC closure rule

$$[\delta^\star_{\hat{\Lambda}_1} \overset{\star}{,} \delta^\star_{\hat{\Lambda}_2}] = \delta^\star_{-i[\hat{\Lambda}_1 \overset{\star}{,} \hat{\Lambda}_2]} \ .$$

consistently generalizes its commutative counterpart.

1. Universal enveloping algebra (UEA) approach ; infinite number of dofs.
2. Seiberg-Witten (SW) map ; induced NC transformations,

$$\delta^\star_{\Lambda} \hat{V}_\mu = \hat{V}_\mu(V_\mu + \delta_\epsilon V_\mu) - \hat{V}_\mu(V_\mu) \ .$$

SW map between NC and classical fields :

$$\hat{\Lambda}_\epsilon = \epsilon - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, \partial_\sigma \epsilon\} + \mathcal{O}(\theta^2) \ ,$$

$$\hat{V}_\mu = V_\mu - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, \partial_\sigma V_\mu + F_{\sigma\mu}\} + \mathcal{O}(\theta^2) \ .$$

AdS algebra

AdS algebra $\mathfrak{so}(2,3)$ has ten generators $M_{AB} = -M_{BA}$ $(A, B = 0, 1, 2, 3, 5)$

$$[M_{AB}, M_{CD}] = i(\eta_{AD}M_{BC} + \eta_{BC}M_{AD} - \eta_{AC}M_{BD} - \eta_{BD}M_{AC})$$

$\eta_{AB} = (+, -, -, -, +)$. Split the generators into six AdS rotations M_{ab} and four AdS translations M_{a5} $(a, b = 0, 1, 2, 3)$ to obtain

$$\begin{aligned}
&[M_{a5}, M_{b5}] = -iM_{ab} \\
&[M_{ab}, M_{c5}] = i(\eta_{bc}M_{a5} - \eta_{ac}M_{b5}) \\
&[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac})
\end{aligned}$$

Introduce $\mathcal{M}_{ab} := M_{ab}$ and $\mathcal{P}_a := l^{-1}M_{a5} = \alpha M_{a5}$, where l is AdS radius and $\alpha = l^{-1}$

$$\begin{aligned}
[\mathcal{P}_a, \mathcal{P}_b] &= -i\alpha^2\mathcal{M}_{ab} \\
[\mathcal{M}_{ab}, \mathcal{P}_c] &= i(\eta_{bc}\mathcal{P}_a - \eta_{ac}\mathcal{P}_b)\ , \\
[\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= i(\eta_{ad}\mathcal{M}_{bc} + \eta_{bc}\mathcal{M}_{ad} - \eta_{ac}\mathcal{M}_{bd} - \eta_{bd}\mathcal{M}_{ac})
\end{aligned}$$

In the limit $\alpha \to 0$ (or $l \to \infty$), AdS algebra $\to$ Poincaré algebra (WI contraction).

$$M_{AB} = \frac{i}{4}[\Gamma_A, \Gamma_B]\ , \qquad \{\Gamma_A, \Gamma_B\} = 2\eta_{AB}\ , \qquad \Gamma_A = (i\gamma_a\gamma_5, \gamma_5)$$

In this particular representation, $M_{ab} = \frac{i}{4}[\gamma_a, \gamma_b] = \frac{1}{2}\sigma_{ab}$ and $M_{a5} = -\frac{1}{2}\gamma_a$.

AdS gauge theory of gravity

AdS gauge field

$$\omega_\mu = \frac{1}{2}\omega_\mu{}^{AB}M_{AB} = \frac{1}{4}\omega_\mu{}^{ab}\sigma_{ab} - \frac{1}{2}\omega_\mu{}^{a5}\gamma_a$$

AdS field strength

$$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu - i[\omega_\mu,\omega_\nu] = \Big(R_{\mu\nu}{}^{ab} - (\omega_\mu{}^{a5}\omega_\nu{}^{b5} - \omega_\mu{}^{b5}\omega_\nu{}^{a5})\Big)\frac{\sigma_{ab}}{4} - F_{\mu\nu}{}^{a5}\frac{\gamma_a}{2}$$

$$\begin{aligned} R_{\mu\nu}{}^{ab} &= \partial_\mu\omega_\nu{}^{ab} - \partial_\nu\omega_\mu{}^{ab} + \omega_\mu{}^a{}_c\,\omega_\nu{}^{cb} - \omega_\nu{}^a{}_c\,\omega_\mu{}^{cb} \\ F_{\mu\nu}{}^{a5} &= D^L_\mu\omega_\nu{}^{a5} - D^L_\nu\omega_\mu{}^{a5} \end{aligned}$$

Gauge parameter $\epsilon = \frac{1}{2}\epsilon^{AB}M_{AB}$

$$\begin{aligned} \delta_\epsilon\omega_\mu{}^{ab} &= \partial_\mu\epsilon^{ab} - \epsilon^a{}_c\,\omega_\mu{}^{cb} + \epsilon^b{}_c\,\omega_\mu{}^{ca} - \epsilon^a{}_5\,\omega_\mu{}^{5b} + \epsilon^b{}_5\,\omega_\mu{}^{5a} \\ \delta_\epsilon\omega_\mu{}^{a5} &= \partial_\mu\epsilon^{a5} - \epsilon^a{}_c\,\omega_\mu{}^{c5} + \epsilon^5{}_c\,\omega_\mu{}^{ca} \\ \delta_\epsilon F_{\mu\nu}{}^{ab} &= -\epsilon^{ac}F_{\mu\nu c}{}^b + \epsilon^{bc}F_{\mu\nu c}{}^a - \epsilon^{a5}F_{\mu\nu 5}{}^b + \epsilon^{b5}F_{\mu\nu 5}{}^a \\ \delta_\epsilon F_{\mu\nu}{}^{a5} &= -\epsilon^{ac}F_{\mu\nu c}{}^5 + \epsilon^{5c}F_{\mu\nu c}{}^a \end{aligned}$$

Set $\epsilon^{a5} = 0$ and identify $\omega_\mu{}^{ab}$ with spin-connection and $\omega_\mu{}^{a5} = e^a_\mu/l$.

AdS gravity action

Introduce an auxiliary field $\phi = \phi^A \Gamma_A$; it is a space-time scalar and internal space 5-vector transforming in the adjoint representation of $SO(2,3)$, that is $\delta_\epsilon \phi = i[\epsilon, \phi]$.

$$S_{AdS} = \frac{il}{64\pi G_N} \int \mathrm{d}^4 x \; \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \phi$$

Auxiliary field is constrained by $\phi^2 = \eta_{AB}\phi^A\phi^B = l^2$. To break $SO(2,3)$ to $SO(1,3)$ set $\phi^a = 0$ and $\phi^5 = l$ (physical gauge)

$$\phi|_{\text{g.f.}} = l\gamma_5 \; .$$

$$S_{AdS}|_{\text{g.f.}} = -\frac{1}{16\pi G_N} \int \mathrm{d}^4 x \left(e\Big(R - \frac{6}{l^2}\Big) + \frac{l^2}{16} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu}{}^{mn} R_{\rho\sigma}{}^{rs} \epsilon_{mnrs} \right)$$

Cosmological constant $\Lambda = -3/l^2$ vanishes under WI contraction.

$SO(2,3)_\star$ model of pure NC gravity

1. NC deformation of GR (NC *Einstein-Hilbert action*) :

$$S_{NC} = \frac{il}{64\pi G_N}\int \mathrm{d}^4x\; \varepsilon^{\mu\nu\rho\sigma}\hat{F}_{\mu\nu}\star\hat{F}_{\rho\sigma}\star\hat{\phi}\;.$$

After SW expansion and symmetry breaking :

$$S_{NC}|_{\text{g.f.}} = -\frac{1}{16\pi G_N}\int \mathrm{d}^4x\,\sqrt{-g}\Bigg(R + \theta^{\alpha\beta}\theta^{\gamma\delta}\Big(\frac{7}{2l^4}R_{\alpha\beta\gamma\delta} - \frac{15}{16l^4}T_{\alpha\beta}{}^{\rho}T_{\gamma\delta\rho} + \dots\Big)\Bigg)\;.$$

2. NC field equations ; deformation of *Minkowski space* ; interpretation of θ-constant noncommutativity ; Fermi inertial coordinates

$$g_{00} = 1 - R_{0m0n}x^m x^n\;,$$

$$g_{0i} = -\frac{2}{3}R_{0min}x^m x^n\;,\qquad g_{ij} = -\delta_{ij} - \frac{1}{3}R_{imjn}x^m x^n\;.$$

M. Dimitrijević Ćirić, B. Nikolić and V. Radovanović, *NC $SO(2,3)_\star$ gravity : noncommutativity as a source of curvature and torsion*, Phys. Rev. D **96**, 064029 (2017)

OSp(4|1) SUGRA

1. Orthosymplectic supergroup $OSp(4|1)$ has 14 generators - 10 AdS generators $\hat{M}_{AB}$ and 4 fermionic generators $\hat{Q}_\alpha$ comprising a single Majorana spinor. Bosonic sector $SO(2,3) \sim Sp(4)$.
2. Supermatrix for the $OSp(4|1)$ gauge field Ω_μ is given by

$$\Omega_\mu = \frac{1}{2}\omega_\mu^{\ AB}\hat{M}_{AB} + \sqrt{\alpha}\bar{\psi}_\mu^\alpha \hat{Q}_\alpha = \left(\begin{array}{c|c} \omega_\mu & \sqrt{\alpha}\psi_\mu \\ \hline \sqrt{\alpha}\bar{\psi}_\mu & 0 \end{array}\right) .$$

3. Action with $OSp(4|1)$ gauge symmetry :

$$S_{41} = \frac{il}{32\pi G_N} \int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}(\mathbb{I}_{5\times 5} - \tfrac{1}{2l^2}\Phi^2)\mathbb{F}_{\rho\sigma}\Phi .$$

Auxiliary field

$$\Phi = \left(\begin{array}{c|c} \frac{1}{4}\pi + i\phi^a\gamma_a\gamma_5 + \phi^5\gamma_5 & \lambda \\ \hline -\bar{\lambda} & \pi \end{array}\right) = |_{\text{g.f.}} \left(\begin{array}{c|c} l\gamma_5 & 0 \\ \hline 0 & 0 \end{array}\right) .$$

In the physical gauge, the action **exactly** reduces to $N = 1$ AdS_4 SUGRA action

$$S_{41}|_{\text{g.f.}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x \left(e\big(R(e,\omega) - 6\alpha^2\big) + 2\varepsilon^{\mu\nu\rho\sigma}\Big(\bar{\psi}_\mu\gamma_5\gamma_\nu\Big(D^L_\rho + \frac{i\alpha}{2}\gamma_\rho\Big)\psi_\sigma\Big)\right) .$$

4. **The leading non-vanishing NC correction is quadratic in $\theta^{\mu\nu}$.**

OSp(4|2) SUGRA

Orthosymplectic group $OSp(4|2)$ has 19 generators - 10 AdS generators $\hat{M}_{AB}$, 8 fermionic generators $\hat{Q}^I_\alpha$ ($\alpha = 1, 2, 3, 4$; $I = 1, 2$) comprising a pair of Majorana spinors, and an additional bosonic generator $\hat{T}$.

$$\Omega_\mu = \hat{\omega}_\mu + \sqrt{\alpha}\bar{\psi}^\alpha_\mu \hat{Q}_\alpha + \alpha \mathcal{A}_\mu \hat{T} = \left(\begin{array}{c|cc} \omega_\mu & \sqrt{\alpha}\psi^1_\mu & \sqrt{\alpha}\psi^2_\mu \\ \hline \sqrt{\alpha}\bar{\psi}^1_\mu & 0 & i\alpha\mathcal{A}_\mu \\ \sqrt{\alpha}\bar{\psi}^2_\mu & -i\alpha\mathcal{A}_\mu & 0 \end{array}\right) .$$

M. Dimitrijević Ćirić, D. Gočanin, N. Konjik and V. Radovanović, “Noncommutative Electrodynamics from $SO(2,3)_\star$ Model of Noncommutative Gravity”, Eur. Phys. J. C 78 (2018) no.7, 548.

Majorana spinors, ψ^1_μ and ψ^2_μ, can be combined into an $SO(2)$ doublet,

$$\Psi_\mu = \begin{pmatrix} \psi^1_\mu \\ \psi^2_\mu \end{pmatrix} .$$

Charged Dirac vector-spinors $\psi^\pm_\mu = \psi^1_\mu \pm i\psi^2_\mu$, related by C-conjugation, $\psi^-_\mu = C\bar{\psi}^{+T}_\mu$.

$$S_{42} = \frac{il}{32\pi G_N}\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}\left(\mathbb{I}_{6\times 6} - \frac{1}{2l^2}\Phi^2\right)\mathbb{F}_{\rho\sigma}\Phi .$$

Gauge-fixed action is not complete !

$$S_{42}|_{\text{g.f.}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x \left(e\Big(R(e,\omega) - 6\alpha^2\Big) + \frac{1}{16\alpha^2} R_{\mu\nu}{}^{mn} R_{\rho\sigma}{}^{rs} \varepsilon^{\mu\nu\rho\sigma}\varepsilon_{mnrs} \right)$$

$$+\,\varepsilon^{\mu\nu\rho\sigma}\Big(2\bar{\Psi}_\mu\gamma_5\gamma_\nu(D_\rho + \alpha\mathcal{A}_\rho i\sigma^2)\Psi_\sigma + i\mathcal{F}_{\mu\nu}(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma) - \frac{i}{2}(\bar{\Psi}_\mu i\sigma^2\Psi_\nu)(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma)\Big)\,.$$

$OSp(4|2)$ field strength (bosonic blocks)

$$\mathbb{F}_{\mu\nu} = \left(\begin{array}{c|cc} \widetilde{F}_{\mu\nu} & * & * \\ \hline * & 0 & i\alpha\widetilde{\mathcal{F}}_{\mu\nu} \\ * & -i\alpha\widetilde{\mathcal{F}}_{\mu\nu} & 0 \end{array}\right),$$

Supplementary action invariant under purely bosonic $SO(2,3)\times U(1)$ sector of $OSp(4|2)$, involving the bosonic field strength
$\widetilde{f}_{\mu\nu} := \widetilde{F}_{\mu\nu} + \widetilde{\mathcal{F}}_{\mu\nu} = \widetilde{F}_{\mu\nu} + (\mathcal{F}_{\mu\nu} - \bar{\Psi}_\mu i\sigma^2\Psi_\nu)\,.$

$$S_A \sim Tr\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma}\left(f\widetilde{f}_{\mu\nu} D_\rho\phi D_\sigma\phi\phi + \frac{i}{6}\, f^2 D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\phi \right) + c.c.$$

Auxiliary field $f = \frac{1}{2} f^{AB} M_{AB}$ (adj. rep. of $SO(2,3)$)

Gauge fixing yields

$$S_A|_{\text{g.f.}} \sim \int \mathrm{d}^4 x \; e\Big(- f^{ab}\widetilde{\mathcal{F}}_{\mu\nu} e^a_\mu e^b_\nu - \frac{1}{2}\, f^{AB} f_{AB}\Big) \; .$$

$$f_{ab} = -\widetilde{\mathcal{F}}_{\mu\nu} e^\mu_a e^\nu_b \; , \quad f_{a5} = 0 \; .$$

Inserting them back we obtain

$$S_A|_{\text{g.f.}} = -\frac{1}{4}\int \mathrm{d}^4 x \; e \, \widetilde{\mathcal{F}}^2 \; .$$

$$\begin{array}{ccc}
(OSp(4|2) \text{ invariant action}) & + & (SO(2,3)\times U(1) \text{ invariant action}) \\
\text{g.f.}\big\downarrow & & \big\downarrow\text{g.f.} \\
(SO(1,3)\times U(1) \text{ invariant action}) & + & (SO(1,3)\times U(1) \text{ invariant action})
\end{array}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{N=2 AdS SUGRA in D=4}}$$

NC deformation

$$S_{42}^{\star} = \frac{il}{32\pi G_N} \int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma} \left(\hat{\mathbb{F}}_{\mu\nu} \star \hat{\mathbb{F}}_{\rho\sigma} \star \hat{\Phi} - \tfrac{1}{2l^2} \hat{\mathbb{F}}_{\mu\nu} \star \hat{\Phi} \star \hat{\Phi} \star \hat{\mathbb{F}}_{\rho\sigma} \star \hat{\Phi} \right) .$$

$$\begin{aligned} S_{42}^{(1)} = \frac{il\theta^{\lambda\tau}}{32\pi G_N} Str \int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma} \Bigg(& -\frac{1}{4}\{\mathbb{F}_{\lambda\tau}, \mathbb{F}_{\mu\nu}\mathbb{F}_{\rho\sigma}\}\Phi + \frac{i}{2}\widehat{D}_\lambda \mathbb{F}_{\mu\nu} \widehat{D}_\tau \mathbb{F}_{\rho\sigma}\Phi \\ & + \frac{1}{2}\{\mathbb{F}_{\lambda\mu}, \mathbb{F}_{\tau\nu}\}\mathbb{F}_{\rho\sigma}\Phi + \frac{1}{2}\mathbb{F}_{\mu\nu}\{\mathbb{F}_{\lambda\rho}, \mathbb{F}_{\tau\sigma}\}\Phi \\ -\frac{1}{2l^2}\Big(& -\frac{1}{4}\{\mathbb{F}_{\lambda\tau}, \mathbb{F}_{\mu\nu}\Phi^2\}\mathbb{F}_{\rho\sigma}\Phi + \frac{i}{2}\widehat{D}_\lambda \mathbb{F}_{\mu\nu} \widehat{D}_\tau \Phi^2 \mathbb{F}_{\rho\sigma}\Phi \\ & + \frac{1}{2}\{\mathbb{F}_{\lambda\mu}, \mathbb{F}_{\tau\nu}\}\Phi^2\mathbb{F}_{\rho\sigma}\Phi + \frac{i}{4}\mathbb{F}_{\mu\nu}[\widehat{D}_\lambda\Phi, \widehat{D}_\tau\Phi]\mathbb{F}_{\rho\sigma}\Phi \\ & + \frac{i}{2}\mathbb{F}_{\mu\nu}\Phi^2\widehat{D}_\lambda \mathbb{F}_{\rho\sigma}\widehat{D}_\tau\Phi + \frac{1}{2}\mathbb{F}_{\mu\nu}\Phi^2\{\mathbb{F}_{\lambda\rho}, \mathbb{F}_{\tau\sigma}\}\Phi\Big)\Bigg) = 0 . \end{aligned}$$

Likewise, we have canonically deformed version of the bosonic action,

$$\begin{aligned} S_A^{\star} = \frac{1}{32l} Tr \int \mathrm{d}^4x\, \varepsilon^{\mu\nu\rho\sigma} \Big(& \hat{f} \star \hat{\bar{f}}_{\mu\nu} \star D_\rho\hat{\phi} \star D_\sigma\hat{\phi} \star \hat{\phi} \\ & + \frac{i}{6}\hat{f} \star \hat{f} \star D_\mu\hat{\phi} \star D_\nu\hat{\phi} \star D_\rho\hat{\phi} \star D_\sigma\hat{\phi} \star \hat{\phi}\Big) + c.c. \end{aligned}$$

NC deformation

$$S_A^{(1)}|_{\text{g.f.}} \neq 0$$

After WI contraction

$$S_A^{(1)}|_{\text{g.f.}} = -\frac{\theta^{\lambda\tau}}{64\kappa}\int \mathrm{d}^4x\ e\ \Big\{ \widetilde{\mathcal{F}}^{\mu\nu} R_{\mu\nu\rho\sigma} R_{\lambda\tau}{}^{\rho\sigma} - \tilde{\mathcal{F}}^{\mu\nu} R_{\rho\sigma\mu\nu} R_{\lambda\tau}{}^{\rho\sigma} - 4\widetilde{\mathcal{F}}^{\mu\rho} R_{\mu\nu\rho\sigma} R_{\lambda\tau}{}^{\nu\sigma}$$
$$- 2\widetilde{\mathcal{F}}^{\mu\nu} R_{\lambda\mu}{}^{\rho\sigma} R_{\tau\nu\rho\sigma} + 8\widetilde{\mathcal{F}}_{\rho\sigma} R_{\lambda\mu}{}^{\mu\rho} R_{\tau\nu}{}^{\nu\sigma} + \widetilde{\mathcal{F}}_{\mu\nu} R_{\lambda\tau}{}^{\mu\nu} R - \frac{4}{\kappa^2}\widetilde{\mathcal{F}}_{\lambda\tau}\widetilde{\mathcal{F}}^2 + \frac{16}{\kappa^2}\widetilde{\mathcal{F}}^{\mu\nu}\widetilde{\mathcal{F}}_{\lambda\mu}\widetilde{\mathcal{F}}_{\tau\nu}$$
$$+ 8(D^L_\lambda R_{\mu\nu}{}^{mc})(D^L_\tau e^r_\rho) e^\sigma_c \left(\widetilde{\mathcal{F}}_\sigma{}^\mu e^\nu_m e^\rho_r - \widetilde{\mathcal{F}}_\sigma{}^\rho e^\mu_r e^\nu_m + \widetilde{\mathcal{F}}_\sigma{}^\nu e^\mu_r e^\rho_m \right)$$
$$+ 2R_{\mu\nu}{}^{ab}\eta_{rs}(D^L_\lambda e^r_\rho)(D^L_\tau e^s_\sigma)\left(\widetilde{\mathcal{F}}^{\mu\nu} e^\rho_a e^\sigma_b + \widetilde{\mathcal{F}}^{\rho\sigma} e^\mu_a e^\nu_b - 4\widetilde{\mathcal{F}}^{\mu\rho} e^\nu_a e^\sigma_b \right) \Big\} .$$

$$S^{(1)}_{\text{low-energy}} = -\frac{9\theta^{\mu\nu}}{16l^4}\int \mathrm{d}^4x\ e\ \widetilde{\mathcal{F}}_{\mu\nu} = -\frac{9\theta^{\mu\nu}}{16l^4}\int \mathrm{d}^4x\ e\ (\mathcal{F}_{\mu\nu} - \bar{\Psi}_\mu i\sigma^2\Psi_\nu)$$
$$= -\frac{9i\theta^{\mu\nu}}{8l^4}\int \mathrm{d}^4x\ e\ (\bar{\psi}^+_\mu \psi^+_\nu) + \text{surface term} .$$

This mass-like term for charged gravitino ψ^+_μ, minimally coupled to gravity, appears due to space-time noncommutativity. The mass-like parameter is $\sim l_P\Lambda^2_{NC}/l^4$.

The end

Thank you !