Noncommutative Scalar Quasinormal modes of RN Black Hole

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Detection of the gravitational waves can help better understanding of structure of space-time

Dominant stage of the perturbed BH are dumped oscillations of the geometry or matter fields (Quasinormal modes)

Noncommutative geometry

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Approaches to NC geometry $\star$-product, NC spectral triple, NC vierbein formalism, matrix models,...

NC space-time from the angular twist

Twist is used to deform a symmetry Hopf algebra
Twist $\mathcal{F}$ is invertible bidifferential operator from the universal enveloping algebra of the symmetry algebra

We work in 4D and deform the space-time by the following twist

$$\mathcal{F} = \mathrm{e}^{-\frac{i}{2}\theta_{ab}X^a \bigotimes X^b}$$
$$[X^a, X^b] = 0, \quad \text{a,b=1,2} \qquad X_1 = \partial_0 \text{ and } X_2 = x\partial_y - y\partial_x$$
$$\mathcal{F} = \mathrm{e}^{\frac{-ia}{2}(\partial_0 \otimes (x\partial_y - y\partial_x) - (x\partial_y - y\partial_x)\otimes\partial_0)}$$

Bilinear maps are deformed by twist!
Bilinear map μ
$\mu : X \times Y \to Z$
$\mu_\star = \mu\mathcal{F}^{-1}$

Commutation relations between coordinates are:

$[\hat{x}^0, \hat{x}] = ia\hat{y},$ All other commutation relations are zero

$[\hat{x}^0, \hat{y}] = -ia\hat{x}$

Our approach: deform space-time by an Abelian twist to obtain commutation relations

Angular twist in curved coordinates $X_1 = \partial_0$ and $X_2 = \partial_\varphi$

-supose that metric tensor $g_{\mu\nu}$ does not depend on t and φ coordinates

-Hodge dual becomes same as in commutative case

Angular noncommutativity

- Product of two plane waves is

$$e^{-ip\cdot x} \star e^{-iq\cdot x} = e^{-i(p+_\star q)\cdot x}$$

where is $p +_\star q = R(q_3)p + R(-p_3)q$ and

$$R(t) \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\left(\frac{at}{2}\right) & \sin\left(\frac{at}{2}\right) & 0 \\ 0 & -\sin\left(\frac{at}{2}\right) & \cos\left(\frac{at}{2}\right) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Angular noncommutativity

- $e^{-ip\cdot x} \star e^{-iq\cdot x} \star e^{-ir\cdot x} = e^{-i(p+_\star q+_\star r)\cdot x}$ gives

$$p +_\star q +_\star r = R(r_3 + q_3)p + R(-p_3 + r_3)q + R(-p_3 - q_3)r$$

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- General case

$$p^{(1)} +_\star \ldots +_\star p^{(N)} = \sum_{j=1}^{N} R\left(-\sum_{1\leq k<j} p_3^{(k)} + \sum_{j<k\leq N} p_3^{(k)}\right) p^{(j)}$$

- Conservation of momentum is broken!

Scalar $U(1)_\star$ gauge theory

If a one-form gauge field $\hat{A} = \hat{A}_\mu \star dx^\mu$ is introduced to the model through a minimal coupling, the relevant action becomes

$$\begin{aligned} S[\hat{\phi}, \hat{A}] &= \int \Big(d\hat{\phi} - i\hat{A} \star \hat{\phi}\Big)^{+} \wedge_\star *_H \Big(d\hat{\phi} - i\hat{A} \star \hat{\phi}\Big) \\ &\quad - \int \frac{\mu^2}{4!} \hat{\phi}^{+} \star \hat{\phi} \epsilon_{abcd}\ e^a \wedge_\star e^b \wedge_\star e^c \wedge_\star e^d \\ &= \int d^4x \sqrt{-g} \star \Big(g^{\mu\nu} \star D_\mu \hat{\phi}^{+} \star D_\nu \hat{\phi} - \mu^2 \hat{\phi}^{+} \star \hat{\phi}\Big) \end{aligned}$$

After expanding action and varying with respect to Φ^+ EOM is

$$g^{\mu\nu}\left(D_\mu D_\nu\phi - \Gamma^\lambda_{\mu\nu}D_\lambda\phi\right) - \frac{1}{4}\theta^{\alpha\beta}g^{\mu\nu}\left(D_\mu(F_{\alpha\beta}D_\nu\phi) - \Gamma^\lambda_{\mu\nu}F_{\alpha\beta}D_\lambda\phi\right.$$
$$\left. -2D_\mu(F_{\alpha\nu}D_\beta\phi) + 2\Gamma^\lambda_{\mu\nu}F_{\alpha\lambda}D_\beta\phi - 2D_\beta(F_{\alpha\mu}D_\nu\phi)\right) = 0$$

Scalar field in the Reissner–Nordström background

RN metric tensor is

$$g_{\mu\nu} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & -\frac{1}{f} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{bmatrix}$$

with $f = 1 - \frac{2MG}{r} + \frac{Q^2 G}{r^2}$ which gives two horizons (r_+ and r_-)
Q-charge of RN BH
M-mass of RN BH
Non-zero components of gauge fields are $A_0 = -\frac{qQ}{r}$ i.e. $F_{r0} = \frac{qQ}{r^2}$
q-charge of scalar field

EOM for scalar field in RN space-time

$$
\Big(\frac{1}{f}\partial_t^2 - \Delta + (1-f)\partial_r^2 + \frac{2MG}{r^2}\partial_r + 2iqQ\frac{1}{rf}\partial_t - \frac{q^2Q^2}{r^2f}\Big)\phi
$$
$$
+\frac{aqQ}{r^3}\Big((\frac{MG}{r} - \frac{GQ^2}{r^2})\partial_\varphi + rf\partial_r\partial_\varphi\Big)\phi = 0
$$

where a is $\theta^{t\varphi}$

Assuming ansatz $\phi_{lm}(t,r,\theta,\varphi) = R_{lm}(r)e^{-i\omega t}Y_l^m(\theta,\varphi)$ we got equation for radial part

$$
fR''_{lm} + \frac{2}{r}(1-\frac{MG}{r})R'_{lm} - \Big(\frac{l(l+1)}{r^2} - \frac{1}{f}(\omega - \frac{qQ}{r})^2\Big)R_{lm}
$$
$$
-ima\frac{qQ}{r^3}\Big((\frac{MG}{r} - \frac{GQ^2}{r^2})R_{lm} + rfR'_{lm}\Big) = 0 \tag{1}
$$

NC QNM solutions

QNM
-special solution of equation
-damped oscillations of a perturbed black hole

A set of the boudary condition which leads to this solution is the following: at the horizon, the QNMs are purely incoming, while in the infinity the QNMs are purely outgoing

Continued fraction method

To get form

$$\frac{d^2\psi}{dy^2} + V\psi = 0$$

y must be

$$y = r + \frac{r_+}{r_+ - r_-}\Big(r_+ - iamqQ\Big)\ln(r - r_+) - \frac{r_-}{r_+ - r_-}\Big(r_- - iamqQ\Big)\ln(r - r_-)$$

y is modified Tortoise RN coordinate

Asymptotic form of the eq. (1)

$$R(r) \to \begin{cases} Z^{out} e^{i\Omega y} y^{-1-i\frac{\omega qQ-\mu^2 M}{\Omega}-amqQ\Omega} & \text{za } y \to \infty \\ \\ Z^{in} e^{-i\left(\omega - \frac{qQ}{r_+}\right)\left(1+iam\frac{qQ}{r_+}\right)y} & \text{za } y \to -\infty \end{cases}$$

Combining assymptotic forms, we get general solution in the form

$$R(r) = e^{i\Omega r}(r - r_-)^{\epsilon} \sum_{n=0}^{\infty} a_n \left(\frac{r - r_+}{r - r_-}\right)^{n+\delta} \tag{2}$$

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$$\delta = -i\frac{r_+^2}{r_+ - r_-}\left(\omega - \frac{qQ}{r_+}\right), \qquad \epsilon = -1 - iqQ\frac{\omega}{\Omega} + i\frac{r_+ + r_-}{2\Omega}\left(\Omega^2 + \omega^2\right),$$

$$\Omega = \sqrt{\omega^2 - \mu^2}$$

Putting general form (2) to eq (1) we get 6-term recurrence relations for a_n:

$$
\begin{aligned}
A_n a_{n+1} + B_n a_n + C_n a_{n-1} + D_n a_{n-2} + E_n a_{n-3} + F_n a_{n-4} = 0, \\
A_3 a_4 + B_3 a_3 + C_3 a_2 + D_3 a_1 + E_3 a_0 = 0, \\
A_2 a_3 + B_2 a_2 + C_2 a_1 + D_2 a_0 = 0, \\
A_1 a_2 + B_1 a_1 + C_1 a_0 = 0, \\
A_0 a_1 + B_0 a_0 = 0,
\end{aligned}
$$

$$
\begin{aligned}
A_n &= r_+^3 \alpha_n, \\
B_n &= r_+^3 \beta_n - iamqQ(r_+ - r_-) r_+ (n+\delta) - \frac{1}{2} iamqQ(r_+ + r_-) r_+ \\
&\quad + iamqQ r_+ r_- - 3 r_+^2 r_- \alpha_{n-1}, \\
C_n &= r_+^3 \gamma_n + 3 r_+ r_-^2 \alpha_{n-2} + iamqQ(r_+ - r_-)(2r_+ + r_-)(n+\delta-1) \\
&\quad - iamqQ(r_+ - r_-) r_+ \epsilon \qquad + \frac{1}{2} iamqQ(r_+ + r_-)(2r_+ + r_-) \\
&\quad - 3 iamqQ r_+ r_- + amqQ\Omega (r_+ - r_-)^2 r_+ - 3 r_+^2 r_- \beta_{n-1} +, \\
D_n &= -r_-^3 \alpha_{n-3} + 3 r_+ r_-^2 \beta_{n-2} - 3 r_+^2 r_- \gamma_{n-1} + iamqQ(r_+^2 - r_-^2)\epsilon + 3 iamqQ r_+ r_- \\
&\quad - amqQ\Omega (r_+ - r_-)^2 r_- - iamqQ(r_+ - r_-)(r_+ + 2r_-)(n+\delta-2) \\
&\quad - \frac{1}{2} iamqQ(r_+ + r_-)(r_+ + 2r_-), \\
E_n &= 3 r_+ r_-^2 \gamma_{n-2} - r_-^3 \beta_{n-3} + iamqQ(r_+ - r_-) r_- (n+\delta-3) \\
&\quad - iamqQ(r_+ - r_-) r_- \epsilon + \frac{1}{2} iamqQ(r_+ + r_-) r_- iamqQ r_+ r_-, \\
F_n &= -r_-^3 \gamma_{n-3},
\end{aligned}
$$

$$\alpha_n = (n+1)\Big[n+1-2i\frac{r_+}{r_+-r_-}(\omega r_+ - qQ)\Big],$$

$$\begin{aligned}\beta_n &= \epsilon + (n+\delta)(2\epsilon - 2n - 2\delta) + 2i\Omega(n+\delta)(r_+ - r_-) - l(l+1) - \mu^2 r_-^2 \\ &+ \frac{2\omega r_-^2}{r_+-r_-}(\omega r_+ - qQ) - \frac{2r_-^2}{(r_+-r_-)^2}(\omega r_+ - qQ)^2 + 4\omega r_-(\omega r_+ - qQ) \\ &- \frac{2r_-}{r_+-r_-}(\omega r_+ - qQ)^2 + (r_+-r_-)\Big[i\Omega + 2\omega(\omega r_+ - qQ) - \mu^2(r_+ + r_-)\Big],\end{aligned}$$

$$\gamma_n = \epsilon^2 + (n+\delta-1)(n+\delta-1-2\epsilon) + \Big(\omega r_- - \frac{r_-}{r_+-r_-}(\omega r_+ - qQ)\Big)^2$$

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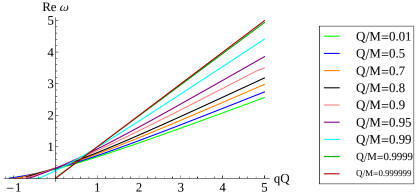
- 3-term relation

$$\alpha_n a_{n+1} + \beta_n a_n + \gamma_n a_{n-1} = 0,$$
$$\alpha_0 a_1 + \beta_0 a_0 = 0$$

gives following equation

$$0 = \beta_0 - \cfrac{\alpha_0\gamma_1}{\beta_1 - \cfrac{\alpha_1\gamma_2}{\beta_2 - \cfrac{\alpha_2\gamma_3}{\beta_3 - \cdots \cfrac{\alpha_n\gamma_{n+1}}{\beta_{n+1} - \cdots}}}}$$


Re ω
qQ
Q/M=0.01
Q/M=0.5
Q/M=0.7
Q/M=0.8
Q/M=0.9
Q/M=0.95
Q/M=0.99
Q/M=0.9999
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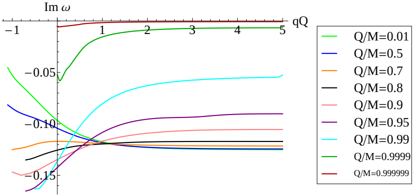
Im ω
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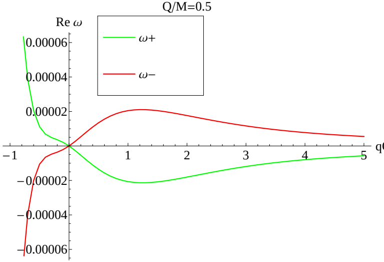
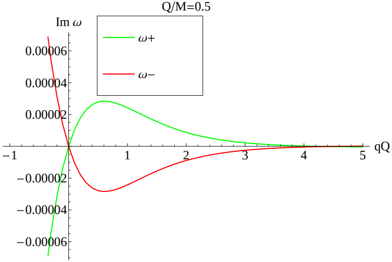

Q/M=0.5
Re ω
qQ
ω+
ω−


Q/M=0.5
Im ω
qQ
ω+
ω−

Outlook

- We constructed Angular twist which induces angular noncommutativity
- Angular NC scalar and vector gauge theory is constructed
- EOM is solved with QNM boundary conditions for scalar field coupled to RN geometry
- But this is toy model!
- Plan for future is to calculate gravitational QNMs and to compare it with results from LIGO, VIRGO, LISA...
- Plan to apply modified momentum conservation law to some measurable process in SM