SLOW-ROLL PARAMETERS IN EXTENDED RSII MODEL

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INTRODUCTION

- The ***inflation theory*** proposes a period of extremely rapid (exponential) expansion of the universe during the very early stage of the universe.
- Inflation is a process in which the dimensions of the universe have increased exponentially at least $e^{60} \approx 10^{26}$ times.
- Although inflationary cosmology has successfully complemented the Standard Model, the process of inflation, in particular its origin, is still largely unknown.

INTRODUCTION

- Over the past 35 years numerous models of inflationary expansion of the universe have been proposed.
- The simplest model of inflation is based on the existence of a single scalar field, which is called ***inflaton.***
- Recent years - a ***lot of evidence*** from WMAP and Planck observations of the CMB
- The most important ways to ***test inflationary cosmological models*** is to compare the computed and measured values of the ***observational parameters.***

OBSERVATIONAL PARAMETERS

- ***Hubble hierarchy (slow-roll) parameters***

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN}, \;\; i \geq 0, \;\; \epsilon_0 \equiv \frac{H_*}{H}$$

Hubble expansion rate at an arbitrarily chosen time

$$\varepsilon_1 = -\frac{\dot{H}}{H^2},$$

$$\varepsilon_2 = 2\varepsilon_1 + \frac{\ddot{H}}{H\dot{H}}.$$

- Duration of inflation $\varepsilon_i \ll 1$

$$N = \ln\frac{a_{end}}{a} = \int_t^{t_{end}} H dt$$

- The end of inflation $\epsilon_i(\varphi_{end}) \approx 1$
- ***Three independent observational parameters***: amplitude of scalar perturbation A_s, tensor-to-scalar ratio r and scalar spectral index n_s

$$r = 16\varepsilon_1$$

$$n_s = 1 - 2\varepsilon_1 - \varepsilon_2$$

At the lowest order in parameters ε_1 and ε_2

OBSERVATIONAL PARAMETERS

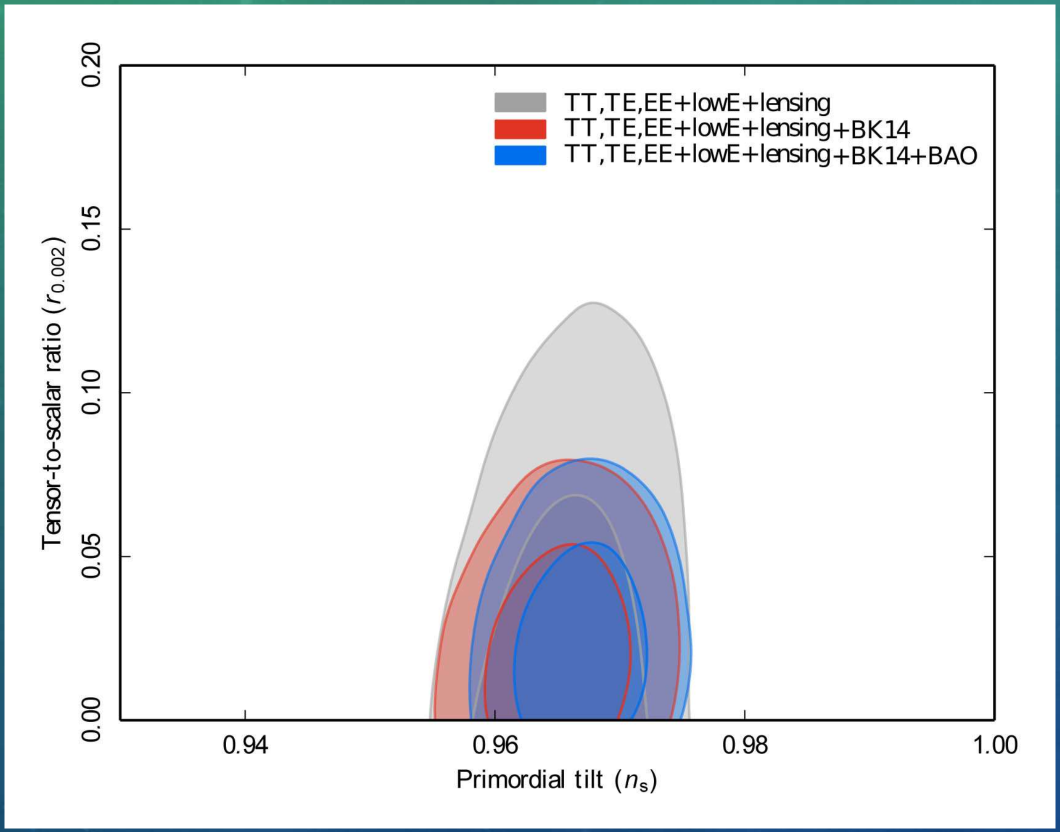
- Scalar spectral index n_s and tensor-to-scalar ratio r for all models up to the second order

$$r = 16\varepsilon_1(1 + C\varepsilon_2 - 2\alpha\varepsilon_1),$$
$$n_s = 1 - 2\varepsilon_1 - \varepsilon_2 - [2\varepsilon_1^2 + (2C + 3 - 2\alpha)\varepsilon_1\varepsilon_2 + C\varepsilon_2\varepsilon_3].$$

- $C \simeq -0{,}72,$
- $\alpha = \frac{1}{6}$ for tachyon inflation in standard cosmology,
- $\alpha = \frac{1}{12}$ for Randall-Sundrum cosmology
- ***Planck*** results ($Planck\ TT,TE,EE{+}lowW{+}lensing{+}BK14$)

$$n_s = 0.9665 \pm 0.0038 \textit{ (68\% CL)},$$
$$r_{0.002} < 0.064 \textit{ (95\% CL)}.$$

OBSERVATIONAL PARAMETERS



- Satellite ***Planck***
  (May 2009 – October 2013)
- ***Planck Collaboration***
  - Latest results - Planck 2018

Planck 2018 results. X Constraints on inflation, arXiv:1807.06211 [astro-ph.CO]

TACHYON INFLATION

- Properties of a tachyon potential

$$V(0) = const, \quad V'(\theta > 0) < 0, \quad V(|\theta| \to \infty) \to 0.$$

- The corresponding Lagrangian and the Hamiltonian are

$$\mathrm{p} \equiv \mathcal{L}(X,\theta) = -V(\theta)\sqrt{1-2X} = -V(\theta)\sqrt{1-\dot{\theta}^2}$$

$$\rho \equiv \mathcal{H} = \frac{V(\theta)}{\sqrt{1-\dot{\theta}^2}}$$

- The Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\mathcal{H} = \frac{8\pi G}{3}\frac{V(\theta)}{\sqrt{1-\dot{\theta}^2}}$$

- M. Milosevic, D.D. Dimitrijevic, G.S. Djordjevic, M.D. Stojanovic, *Dynamics of tachyon fields and inflation - comparison of analytical and numerical results with observation*, Serbian Astronomical Journal. 192 (2016)
- D. Steer, F. Vernizzi, Tachyon inflation: *Tests and comparison with single scalar field inflation*, Phys. Rev. D. 70 (2004) 43527.

DYNAMICS OF INFLATION

1. The energy-momentum conservation equation

$$\dot{\mathcal{H}} = -3H(\mathcal{L} + \mathcal{H}).$$

2. The Hamilton's equations

$$\dot{\theta} = \frac{\partial \mathcal{H}}{\partial \pi_\theta},$$

$$\dot{\pi}_\theta + 3H\pi_\theta = -\frac{\partial \mathcal{H}}{\partial \theta},$$

$\pi_\theta = \frac{\partial \mathcal{L}}{\partial \dot{\theta}}$ is the conjugate momentum and the Hamiltonian $\mathcal{H} = \dot{\theta}\pi_\theta - \mathcal{L}$

TACHYON INFLATION

$$\mathrm{t} = \tau/k,\ \Theta = \mathrm{k}\theta,\ h = \frac{H}{k}.$$

- Dimensionless equations

$$\frac{\ddot{\theta}}{1-\dot{\theta}^2} + 3h\dot{\theta} + \frac{1}{V}\frac{\partial V}{\partial \theta} = 0$$

$$h^2 = \frac{\kappa^2}{3}\rho = \frac{\kappa^2}{3}\frac{V}{\sqrt{1-\dot{\theta}^2}}$$

- Dimensionless constant $\kappa^2 = 8\pi G\lambda k^2$, a choice of a constant σ (brane tension) was motivated by string theory

$$\sigma = \lambda k^2 = \frac{M_s^4}{g_s(2\pi)^3}.$$

CONDITIONS FOR TACHYON INFLATION

- General condition for inflation

$$\frac{\ddot{a}}{a} \equiv \tilde{H}^2 + \dot{\tilde{H}} = \frac{\kappa^2}{3} \frac{\tilde{V}(\Theta)}{\sqrt{1 - \dot{\Theta}^2}} \left(1 - \frac{3}{2}\dot{\Theta}^2\right) > 0.$$

- Slow-roll conditions

$$\ddot{\Theta} \ll 3\tilde{H}\dot{\Theta}, \;\; \dot{\Theta}^2 \ll 1.$$

INITIAL CONDITION FOR TACHYON INFLATION

- Slow-roll parameters

$$\epsilon_1 \simeq \frac{1}{2\kappa^2}\frac{\tilde{V}'^2}{\tilde{V}^3}, \;\; \epsilon_2 \simeq \frac{1}{\kappa^2}\left(-2\frac{\tilde{V}''}{\tilde{V}^2} + 3\frac{\tilde{V}'^2}{\tilde{V}^3}\right).$$

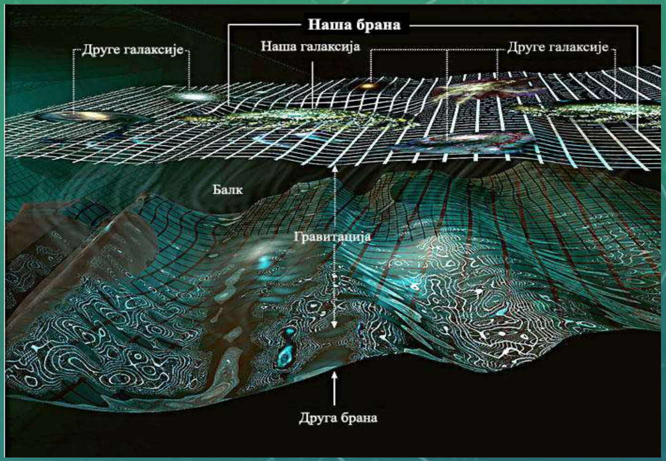
- Number of e-folds

$$N = \kappa \int_{\Theta_i}^{\Theta_e} \frac{\tilde{V}(\Theta)^2}{|\tilde{V}'(\Theta)|} d\Theta$$

$$\Theta_i = \Theta(\tau_i)$$

$$\Theta_e = \Theta(\tau_e)$$

BRANEWORLD COSMOLOGY



- Braneworld universe is based on the scenario in which matter is confined on a brane moving in the higher dimensional bulk with only gravity allowed to propagate in the bulk.
- One of the simplest models - Randall-Sundrum (RS)
- Two branes with opposite tensions are placed at some distance in 5 dimensional space

- N. Bilic, D.D. Dimitrijevic, G.S. Djordjevic, M. Milosevic, *Tachyon inflation in an AdS braneworld with back-reaction*, International Journal of Modern Physics A. 32 (2017) 1750039.
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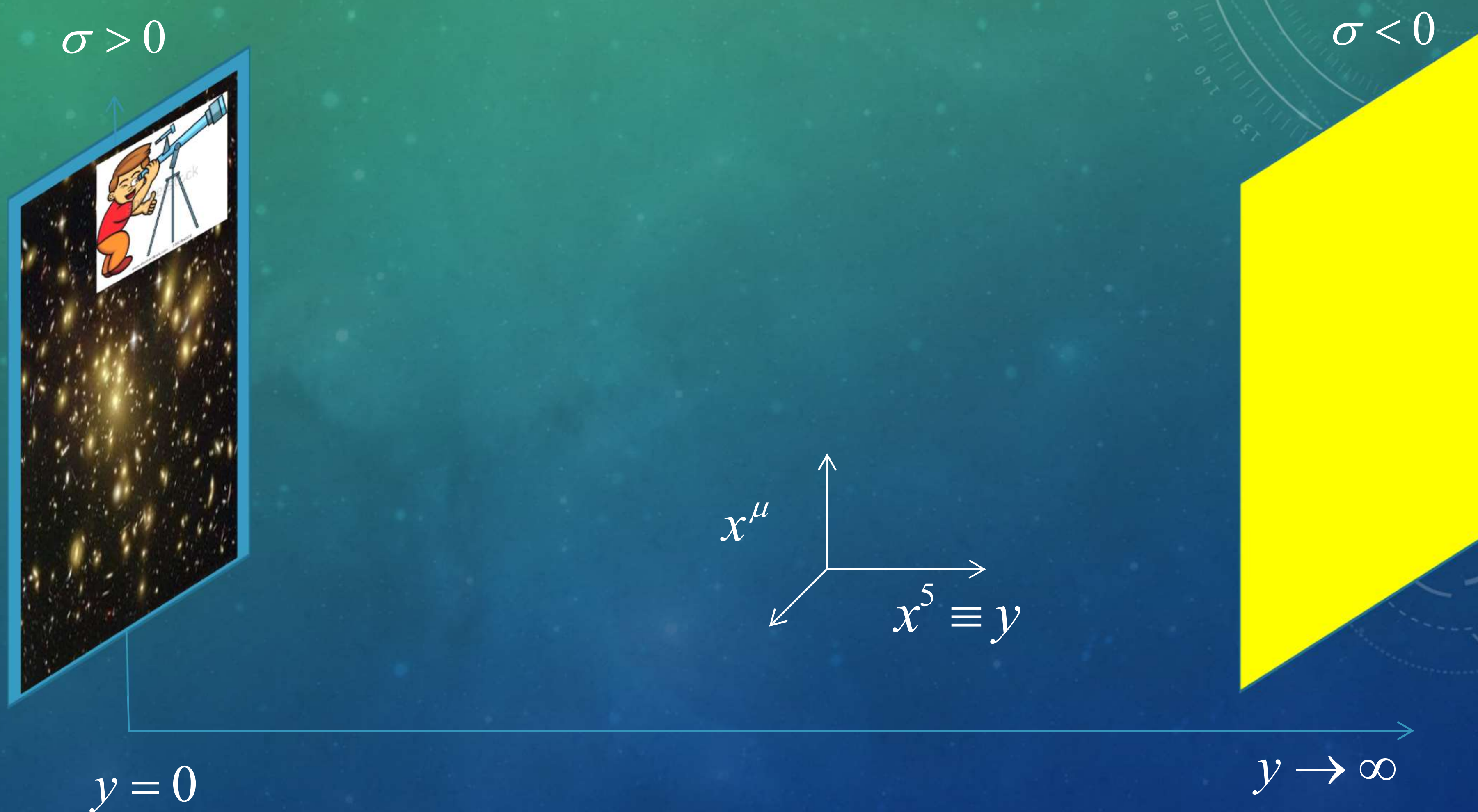
THE RSII MODEL

RSII model – observer is placed on the positive tension brane, 2nd brane is pushed to infinity



THE RSII MODEL

- The space is described by Anti de Siter metric

$$ds^2_{(5)} = e^{-2ky} g^{\mu\nu} dx^\mu dx^\nu - dy^2,$$

where $k = \frac{1}{\ell}$ is the inverse of AdS$_5$ curvature radius

- The Lagrangian and the Hamiltonian

$$\mathcal{L} \equiv p = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - \frac{\lambda\psi^2}{\theta^4} \sqrt{1 - \frac{g^{\mu\nu} \theta_{,\mu} \theta_{,\nu}}{\psi^3}},$$

$$\mathcal{H} \equiv \rho = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} + \frac{\lambda\psi^2}{\theta^4} \left(1 - \frac{g^{\mu\nu} \theta_{,\mu} \theta_{,\nu}}{\psi^3}\right)^{-1/2},$$

where $\psi = 1 + \theta^2 \eta$ and $\eta = \sinh^2\left(\sqrt{4/3\ \pi G}\phi\right)$.

- One additional 3-brane moving in the AdS$_5$ bulk behaves effectively as a tachyon with the potential $V(\theta) \propto \theta^{-4}$

THE RSII MODEL

- The Hamilton's equations

$$\dot{\phi} = \frac{\partial \mathcal{H}}{\partial \pi_\phi}, \qquad \dot{\pi}_\phi + 3H\pi_\phi = -\frac{\partial \mathcal{H}}{\partial \phi},$$

$$\dot{\theta} = \frac{\partial \mathcal{H}}{\partial \pi_\theta}, \qquad \dot{\pi}_\theta + 3H\pi_\theta = -\frac{\partial \mathcal{H}}{\partial \theta}.$$

- The conjugate momenta: $\pi_\phi^\mu \equiv \frac{\partial \mathcal{L}}{\partial \phi_{,\mu}}, \;\; \pi_\theta^\mu \equiv \frac{\partial \mathcal{L}}{\partial \theta_{,\mu}}.$
- The modified Friedman equations

$$H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3}\mathcal{H}\left(1 + \frac{2\pi G}{3k^2}\mathcal{H}\right)},$$

$$\dot{H} = -4\pi G(\mathcal{H} + \mathcal{L})\left(1 + \frac{4\pi G}{3k^2}\mathcal{H}\right)$$

INITIAL CONDITIONS FOR RSII MODEL

- Initial conditions – from a model without radion field
- “Pure” tachyon potential $V(\Theta) = \frac{\lambda}{\Theta^4}$
- Hamiltonian $\mathcal{H} = \frac{\lambda}{\Theta^4}\sqrt{1 + \Pi_\Theta^2\, \Theta^8/\lambda^2}$.
- Dimensionless equations

$$\dot{\theta} = \frac{\theta^4 \pi_\theta}{\sqrt{1+\theta^8\pi_\theta^2}}$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{4}{\theta^5\sqrt{1+\theta^8\pi_\theta^2}}.$$

EXTENDED RSII MODEL

- The RSII model is extended to include ***matter in the bulk.***
- The presence of matter modifies the warp factor which results in two effects:
  - a modification of the RSII cosmology
  - a modification of the tachyon potential.

- N. Bilić, S. Domazet, G.S. Djordjevic, *Particle Creation and Reheating in a Braneworld Inflationary Scenario*, Physical Review D, 96 (2017), 083518
- N. Bilić, S. Domazet, and G. S. Djordjevic, *Tachyon with an inverse power-law potential in a braneworld cosmology*, Class. Quantum Gravity 34, 165006 (2017).

EXTENDED RSII MODEL

- Similar setup as the RSII model, however a more general tachyon potential:

$$V(\Theta) = \frac{\sigma}{\chi^4(\Theta)}$$

- The modified Friedmann equations

$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\chi_{,\varphi} + \frac{\kappa^2}{12}\rho\right)} \qquad \chi_{,\theta} = \frac{\partial\chi}{\partial\theta}$$

$$\dot{h} = -\frac{\kappa^2}{2}(\rho + p)\left(\chi_{,\theta} + \frac{\kappa^2}{6}\rho\right) + \frac{\kappa^2\rho}{6h}\chi_{,\theta\theta}\dot{\theta}.$$

NUMERICAL COMPUTATION

- Calculation:
  - Programming language: C/C++
  - GNU Scientific Library
- Visualization and data analysis
  - Programming language: R

THE SOFTWARE

The model

```
if (MODEL == 2) {
    eta = pow(sinh(sqrt(k*k/6.)*y[0]), 2);
    psi = 1 + y[1]*y[1]*eta;
    koren = sqrt(1 + pow(y[1], 8) * y[3]*y[3]/psi);

    psi2 = psi * psi;
    rho = 1./2. * y[2]*y[2] + psi2/pow(y[1], 4) * koren;
    h = sqrt(k*k/3. * rho * (1 + k*k*rho/12));

    deta = sqrt(k*k/6.) * sinh(sqrt(2.*k*k/3.) * y[0]);

    f[0] = y[2];
    f[1] = pow(y[1],4) / koren * psi * y[3];
    f[2] = -3*h*y[2] - psi/2./pow(y[1],2) * (4 + 3 * pow(y[1],8) * y[3]*y[3] / psi) * deta / koren;
    f[3] = -3*h*y[3] + psi/pow(y[1],5) * (4 - 3 * pow(y[1],10)*eta*y[3]*y[3] / psi) / koren;
    f[4] = h;
    f[5] = h * y[5];
}
```

The initial conditions

```
if (MODEL == 2) {
    f[0] = phi0;
    if (INTC == 2) f[1] = find_initial(N, k);
        else
            if (INTC == 3) f[1] = rnd(0,1);
                else f[1] = pow(k*k*k*k/288/(N+1), 1.0/6.0);
    f[2] = 0.0;
    f[3] = 0.0;
    f[4] = 0.0;
    f[5] = 1.0;
    printf("N = %f\tk = %f\tphi0 = %f\t theta0 = %f\n", N, k, f[0], f[1]);
    fprintf(flog, "N = %f\tk = %f\tphi0 = %f\t theta0 = %f\n", N, k, f[0], f[1]);
}
```

$$\phi \rightarrow f[0]$$
$$\theta \rightarrow f[1]$$
$$\pi_\phi \rightarrow f[2]$$
$$\pi_\theta \rightarrow f[3]$$
$$N \rightarrow f[4]$$
$$a \rightarrow f[5]$$

THE SOFTWARE

init.conf

Test mode = 0
IZLAZ = 0
MCM = 1
MODEL = 2
INTC = 0
NoMAX = 10000

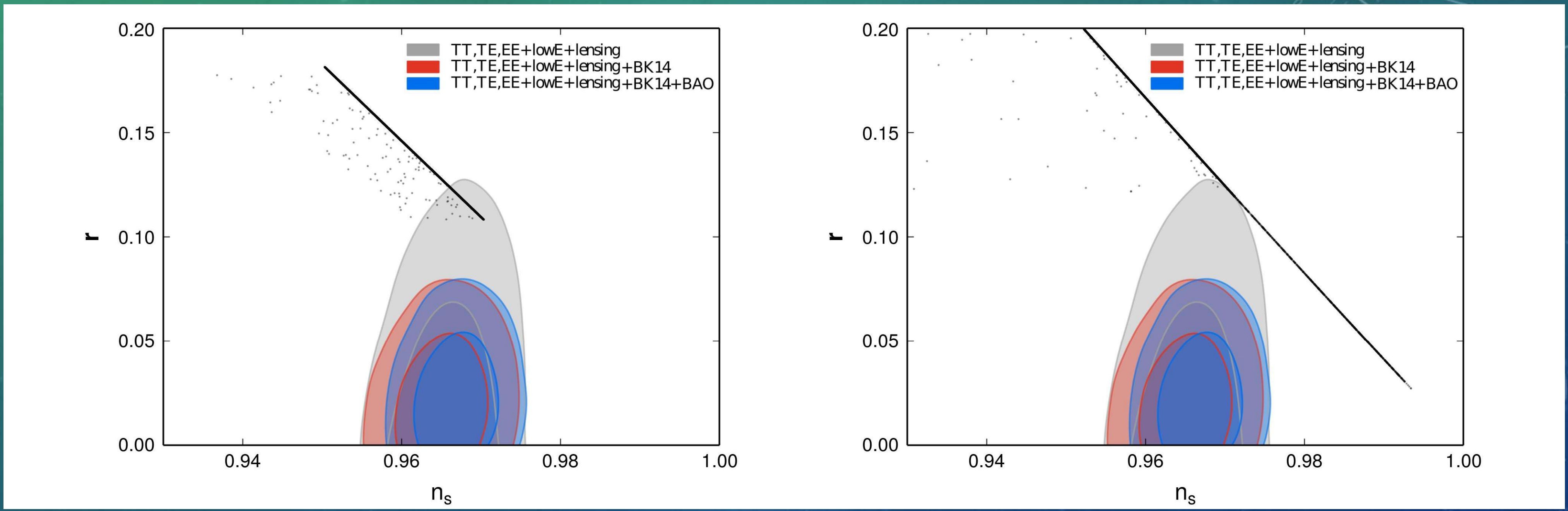
Nmin =	45.00	Nmax =	120.00
kmin =	1.00	kmax =	12.00
phimin =	0.00	phimax =	1.00

MODELS

- Tachyon inflation
- RSII model
- Extended RSII models

- Potentials:
  - $V(\theta) = e^{-\theta}$
  - $V(\theta) = \frac{1}{\theta^4}$
  - $V(\theta) = \frac{1}{\cosh\theta}$

THE TACHYION POTENTIAL $V(\theta) = e^{-\theta}$

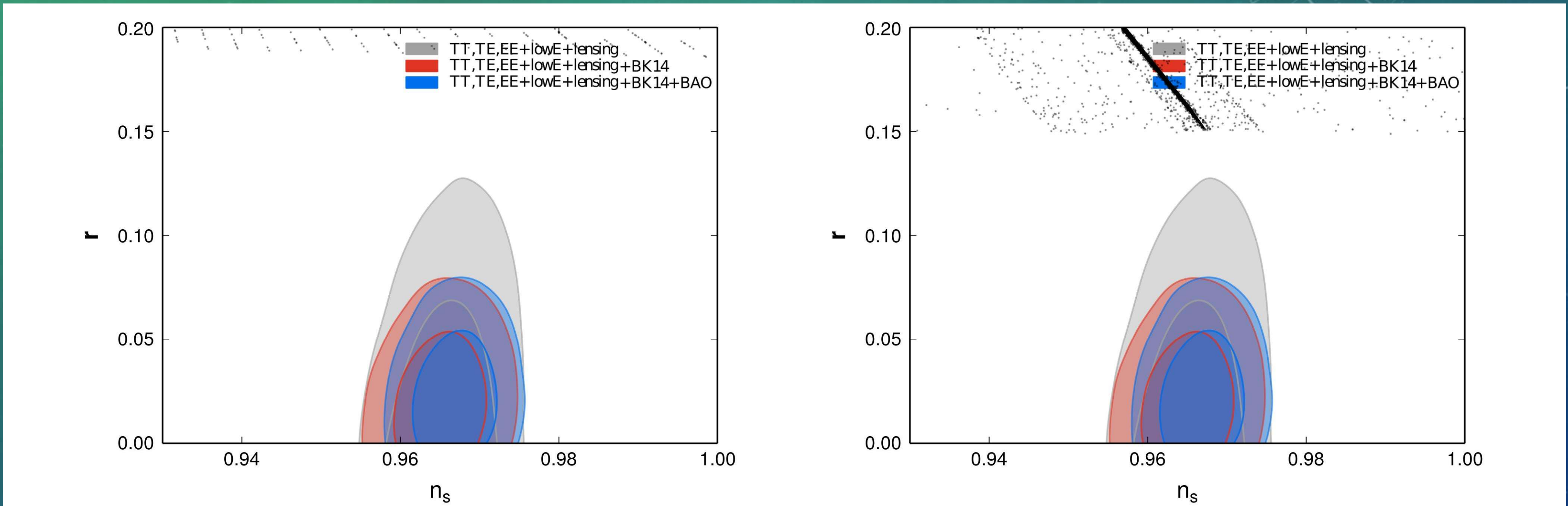


$$h = \sqrt{\frac{\kappa^2}{3}\rho}$$

$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(1 + \frac{\kappa^2}{12}\rho\right)}$$

$45 \leq N < 75,\ 1 \leq \kappa < 10$

THE TACHYION POTENTIAL $V(\theta) = \frac{1}{\theta^4}$



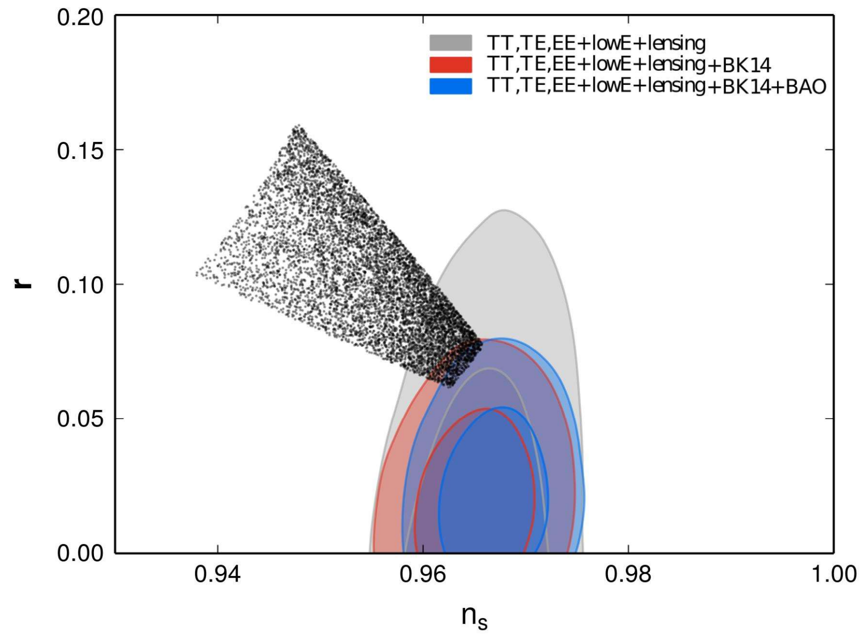
$$h = \sqrt{\frac{\kappa^2}{3}\rho}$$

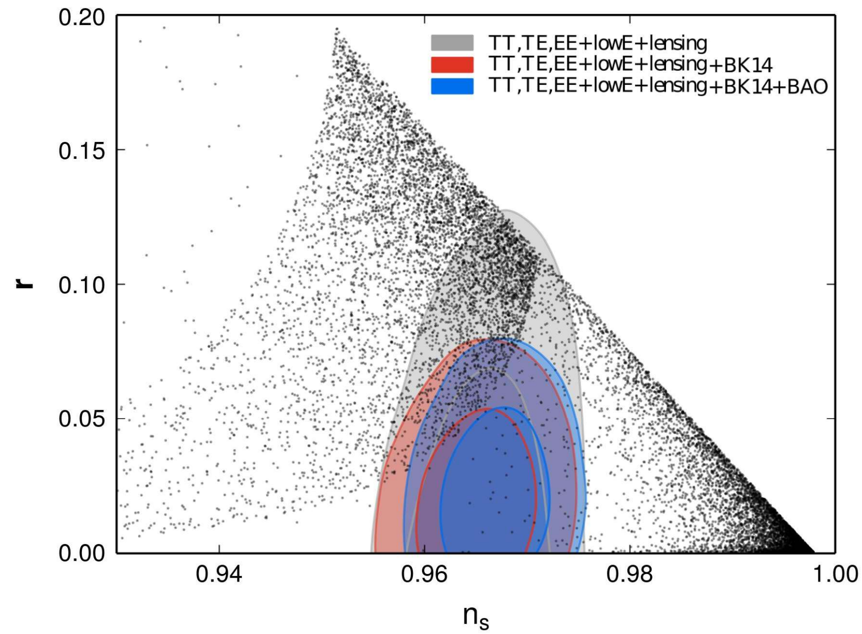
$45 \leq N < 90,\ 1 \leq \kappa < 10$

$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(1 + \frac{\kappa^2}{12}\rho\right)}$$

$45 \leq N < 75,\ 1 \leq \kappa < 10$

THE TACHYION POTENTIAL $V(\theta) = \dfrac{1}{\cosh\theta}$



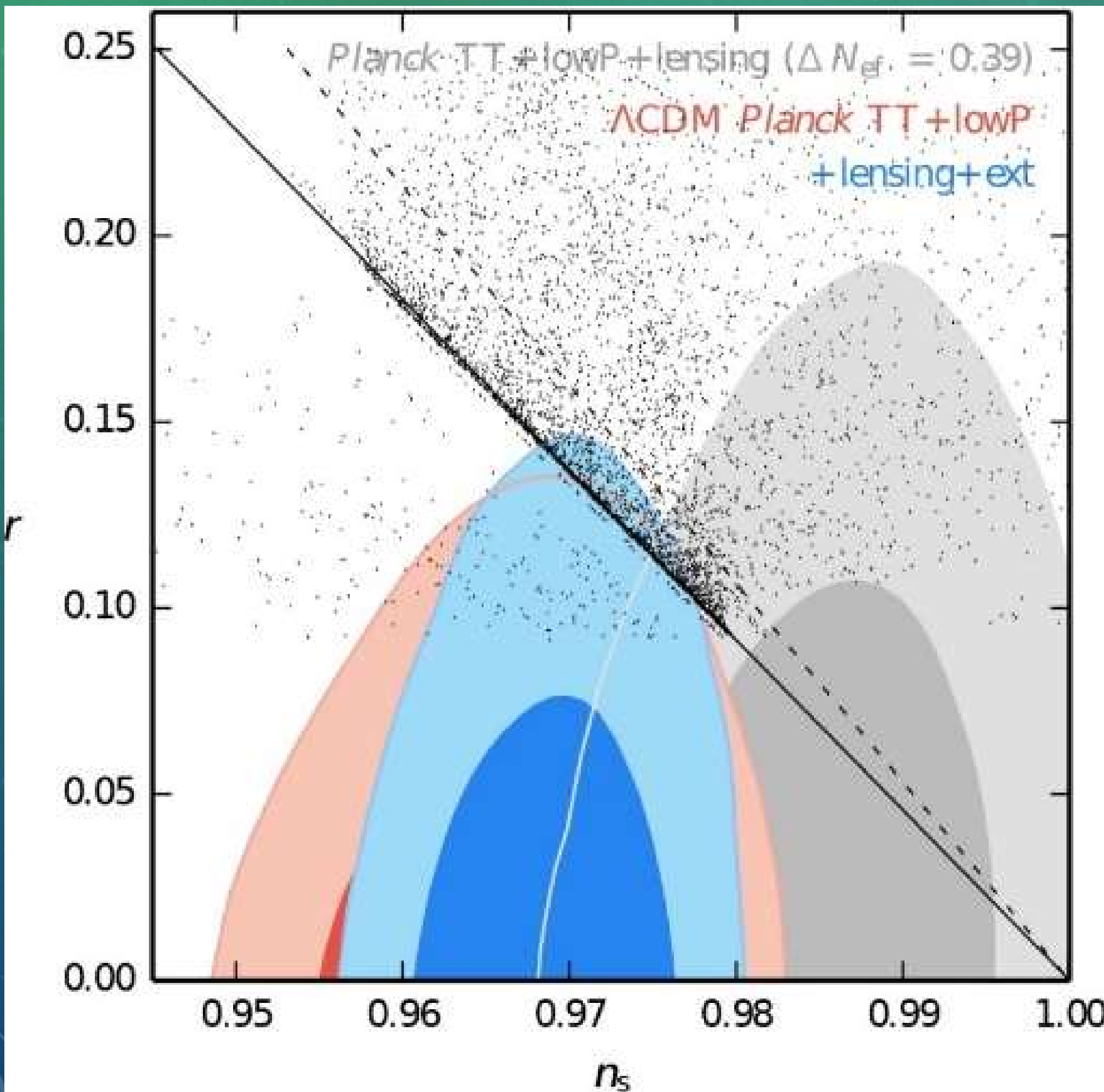


$$h = \sqrt{\frac{\kappa^2}{3}\rho}$$

$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(1 + \frac{\kappa^2}{12}\rho\right)}$$

$45 \leq N < 75,\ 1 \leq \kappa < 10$

RSII MODEL, $V(\theta) = \frac{1}{\theta^4}$



- Free parameters from the interval:

$$60 \le N \le 120$$

$$1 \le \kappa \le 12$$

$$0 \le \phi_0 \le 0,5$$

- Approximate relation:
  - RS model

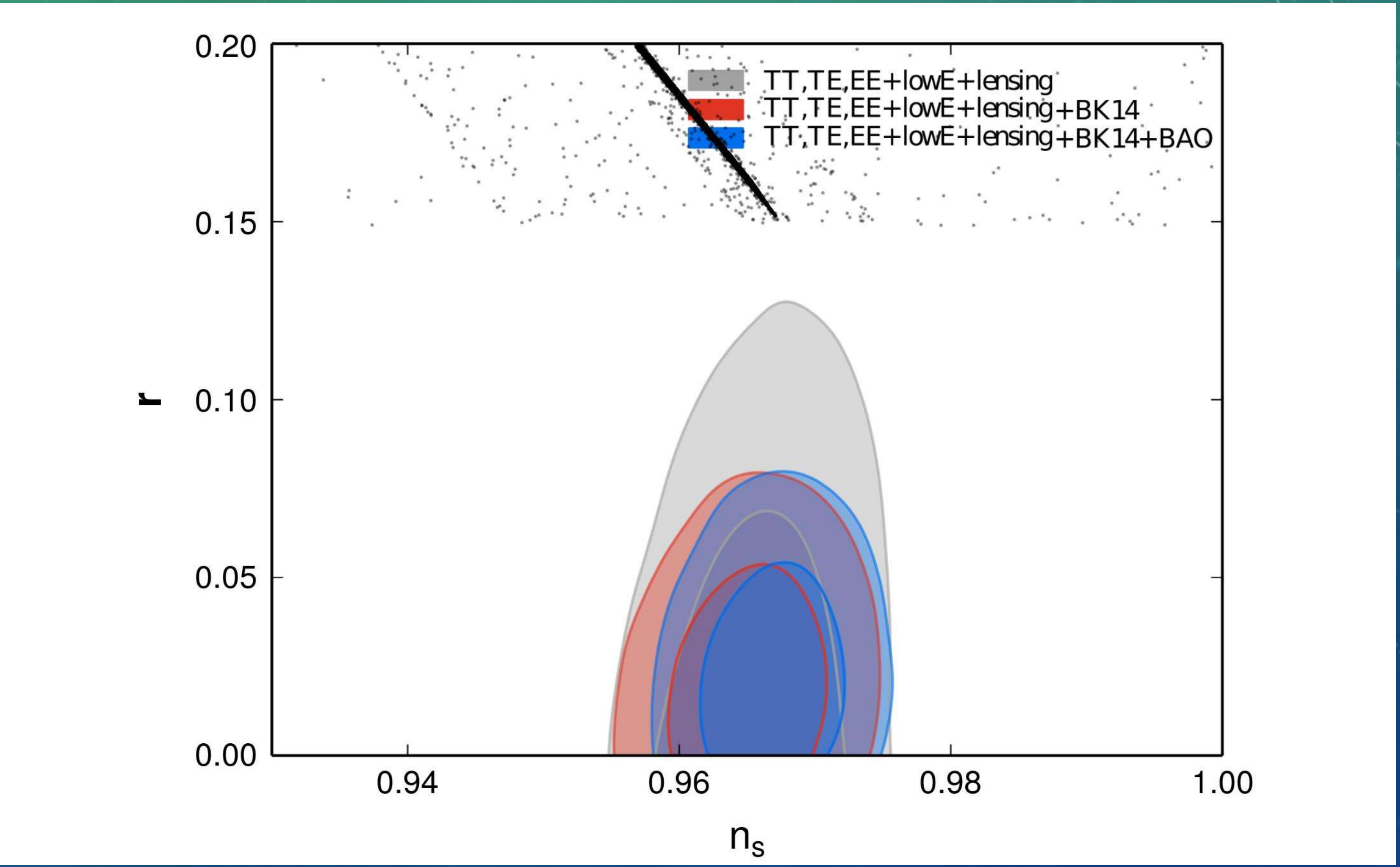
$$r = \frac{32}{7}(1 - n_s)$$

  - Tachyon model (FRW)

$$r = \frac{16}{3}(1 - n_s)$$

Planck 2015 results. XX Constraints on inflation, 2016, A&A, 594, A20

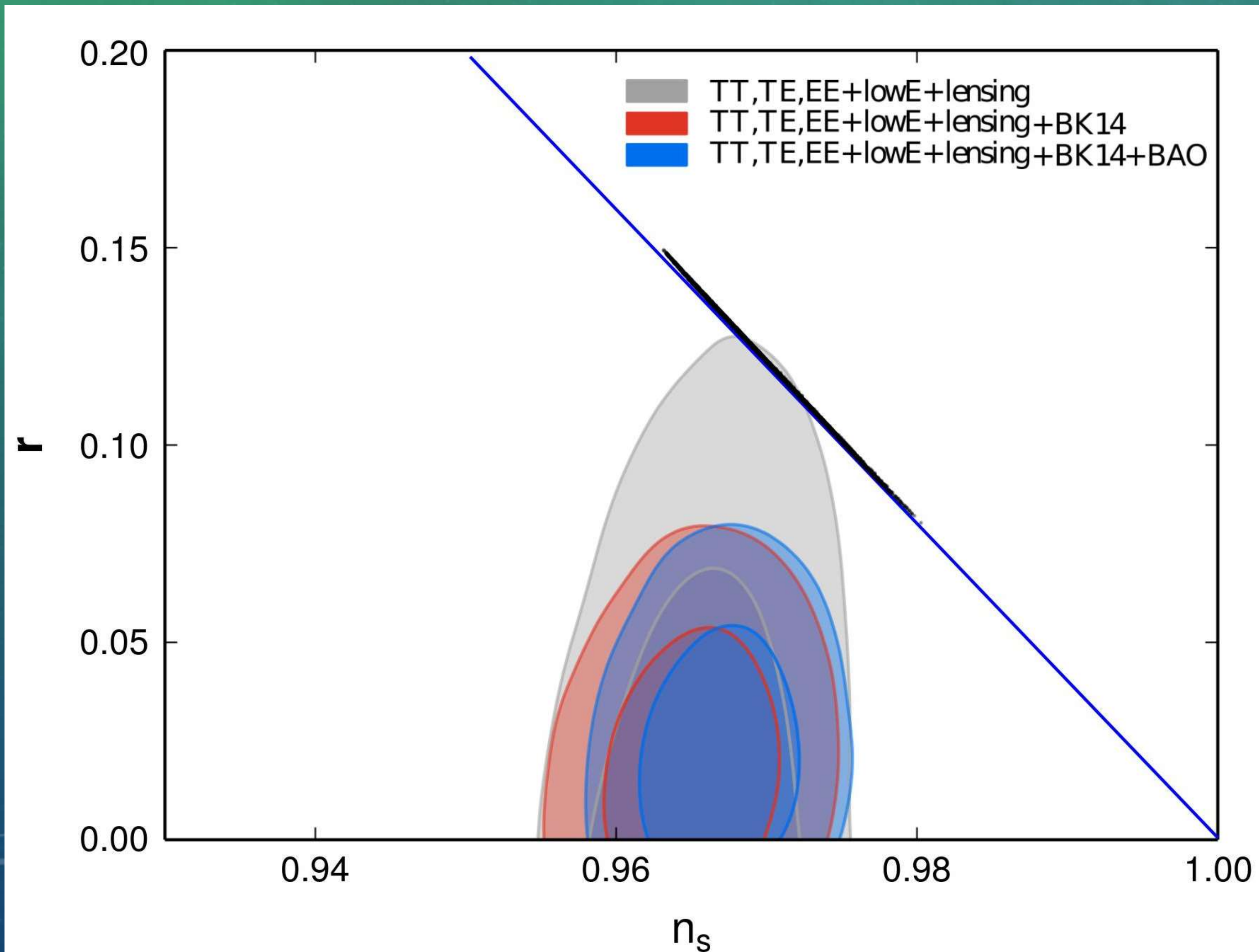
RSII MODEL, $V(\theta) = \frac{1}{\theta^4}$



$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(1 + \frac{\kappa^2}{12}\rho\right)}$$

$45 \leq N \leq 75,\ 1 \leq \kappa \leq 10$

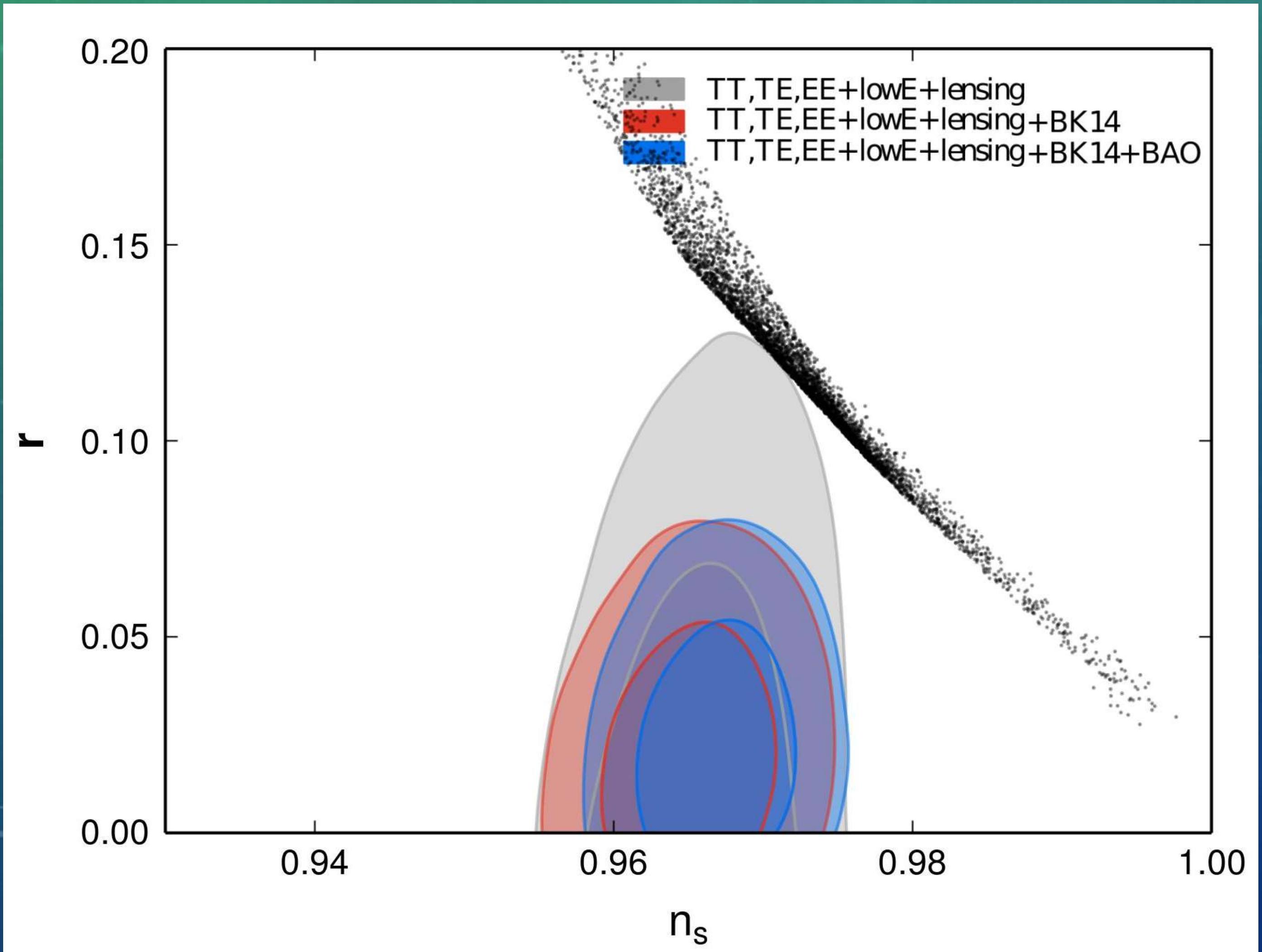
EXTENDED RSII, $V(\theta) = e^{-\theta}$



$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\frac{\partial\chi}{\partial\theta} + \frac{\kappa^2}{12}\rho\right)}$$

$60 \leq N < 90,\ 1 \leq \kappa < 10$

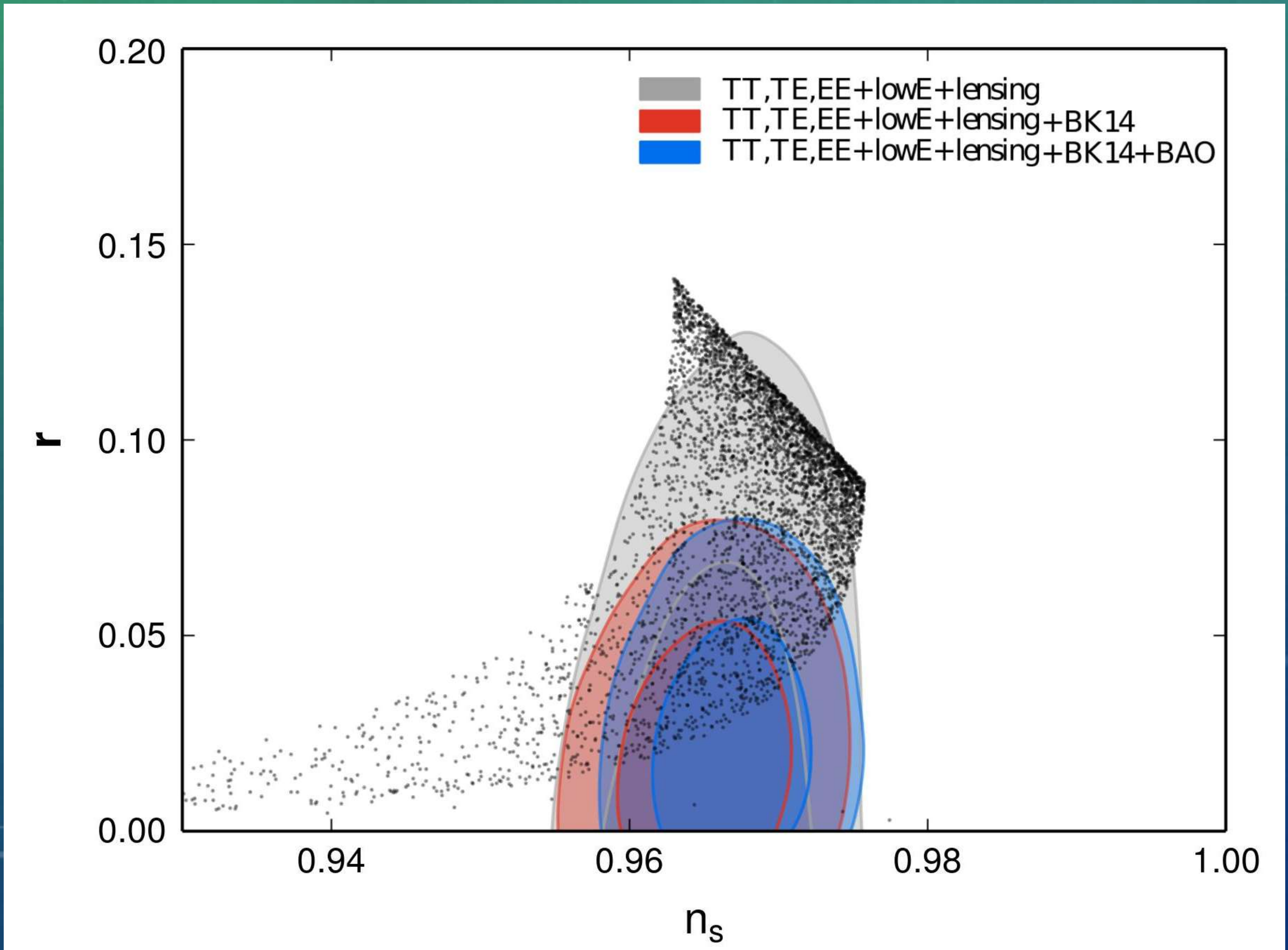
EXTENDED RSII, $V(\theta) = \frac{1}{\theta^{4n}}$



$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\frac{\partial\chi}{\partial\theta} + \frac{\kappa^2}{12}\rho\right)}$$

$60 \leq \mathrm{N} < 90,\ 1 \leq \kappa < 10,$
$0.5 \leq n < 5$

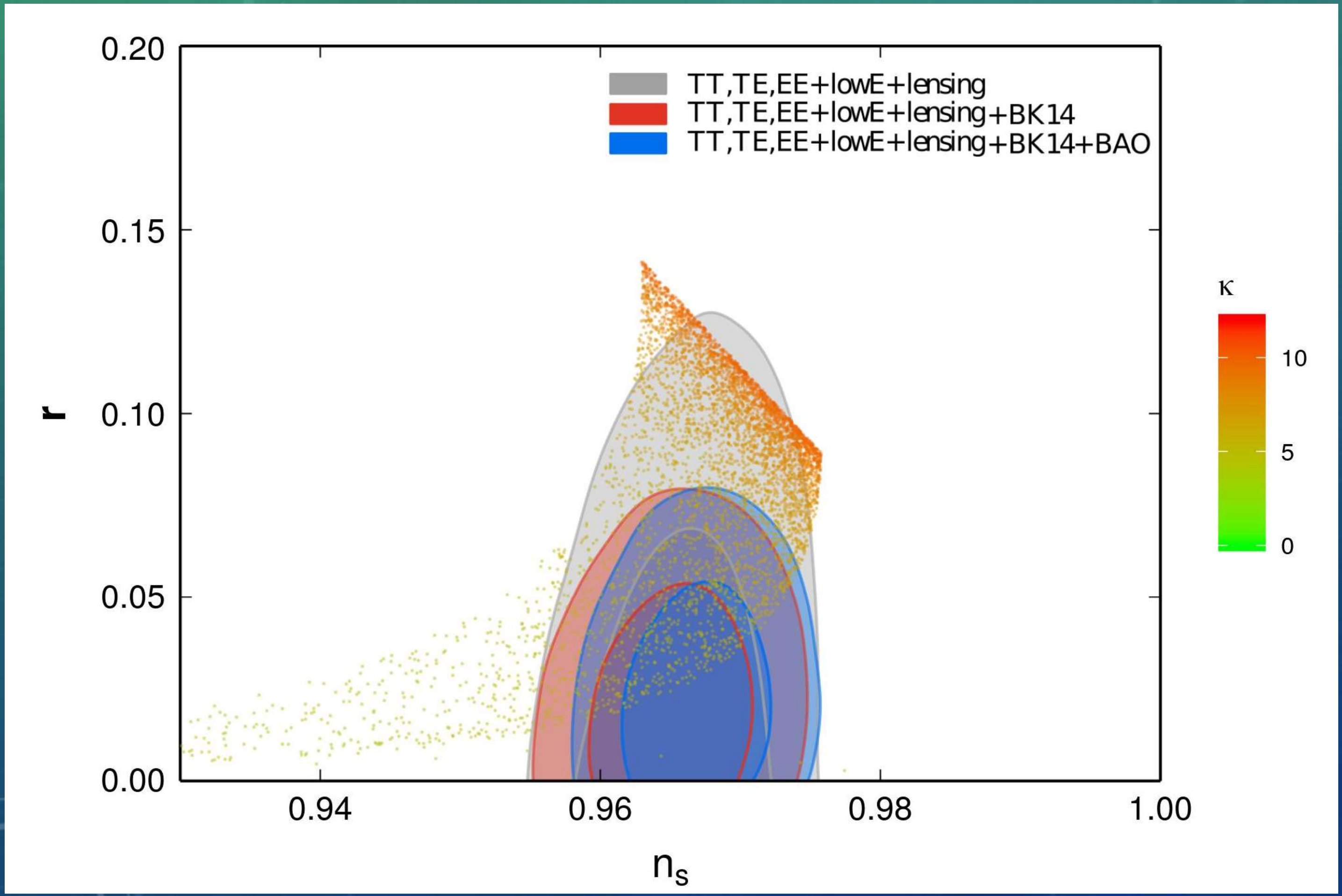
EXTENDED RSII, $V(\theta) = \frac{1}{\cosh\theta}$



$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\frac{\partial\chi}{\partial\theta} + \frac{\kappa^2}{12}\rho\right)}$$

$60 \leq N < 90,\ 1 \leq \kappa < 10$

EXTENDED RSII, $V(\theta) = \dfrac{1}{\cosh\theta}$



$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\frac{\partial\chi}{\partial\theta} + \frac{\kappa^2}{12}\rho\right)}$$

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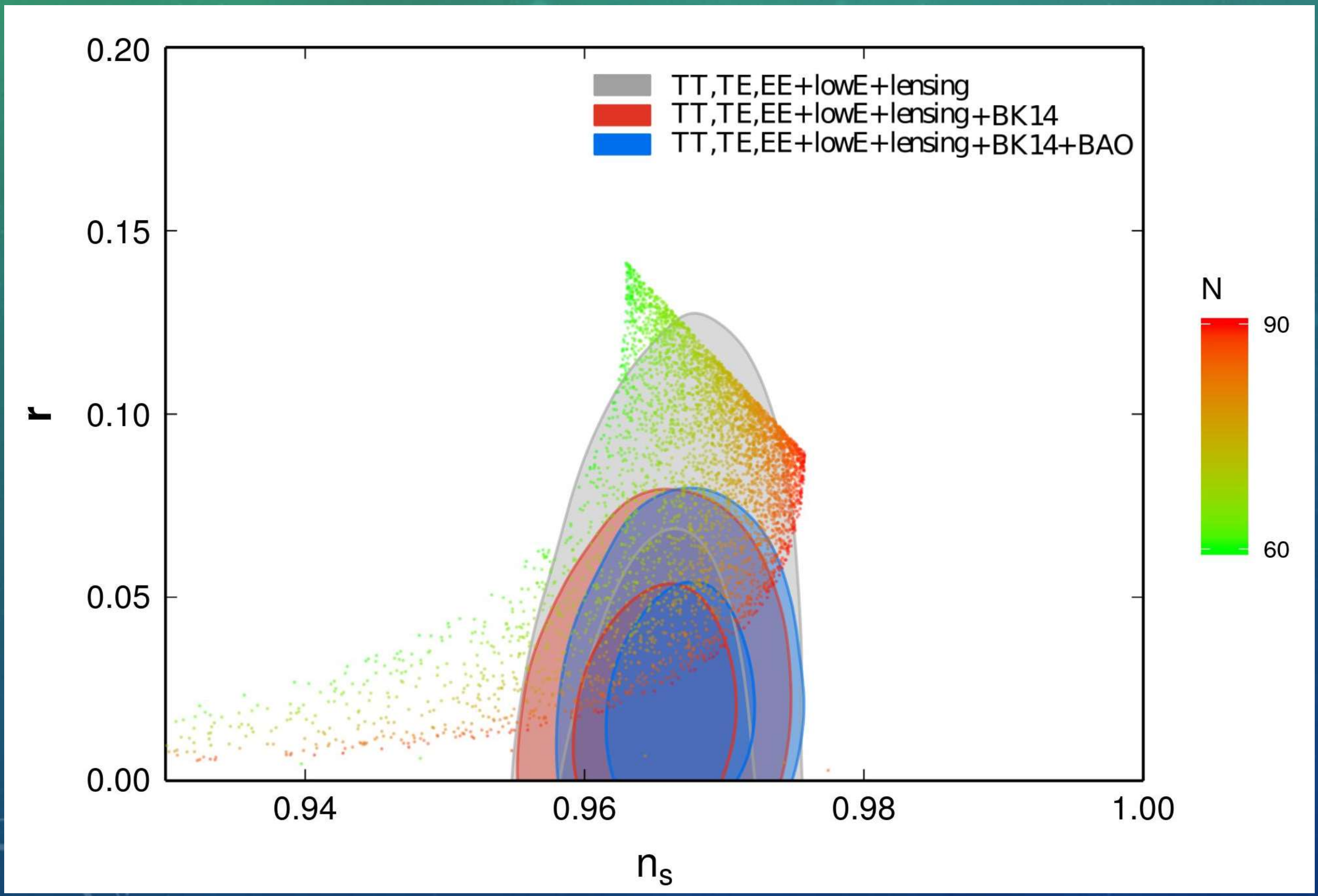
EXTENDED RSII, $V(\theta) = \dfrac{1}{\cosh\theta}$

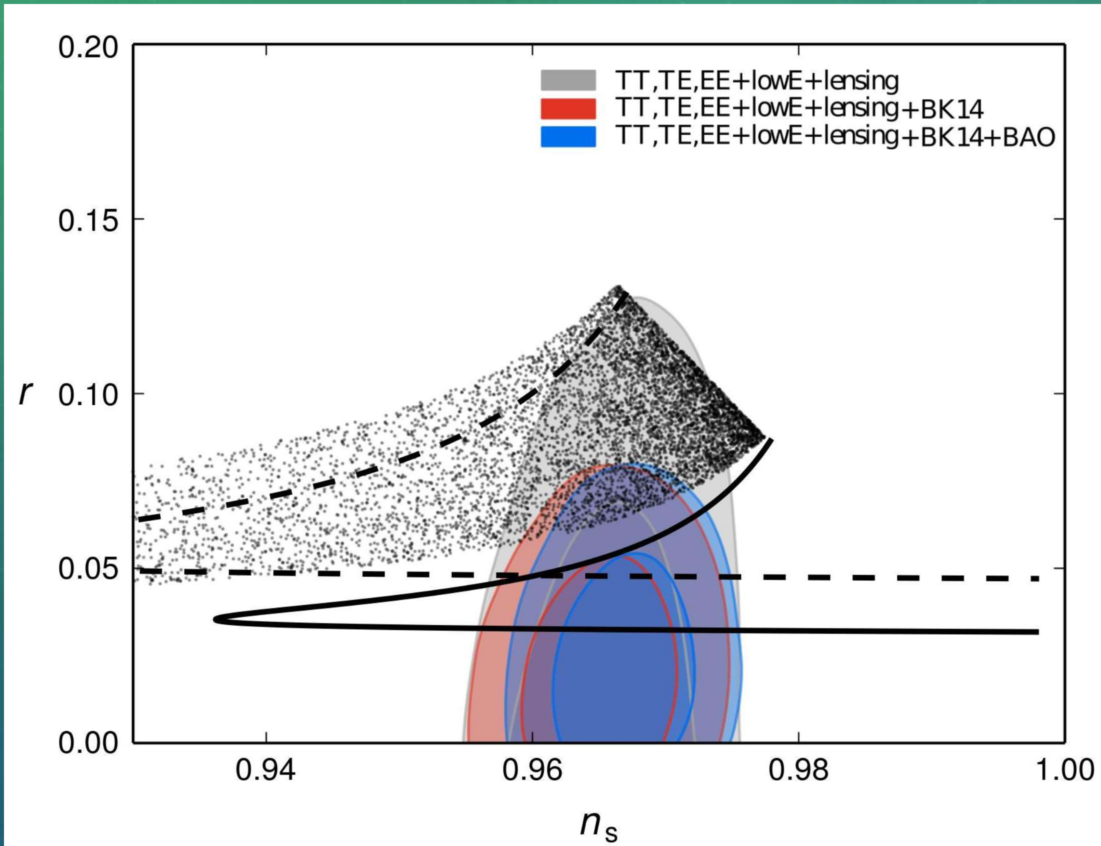


$$h = \sqrt{\frac{\kappa^2}{3}\rho\left(\frac{\partial\chi}{\partial\theta} + \frac{\kappa^2}{12}\rho\right)}$$

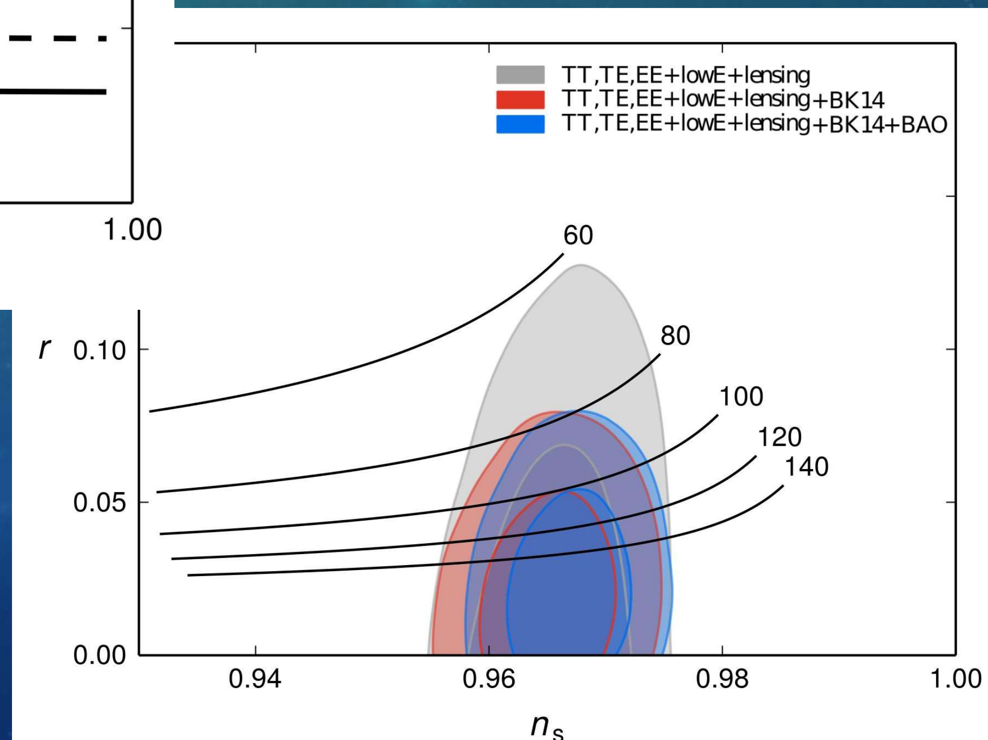
$60 \leq N < 90,\ 1 \leq \kappa < 10$

HOLOGRAFIC COSMOLOGY

RESEARCH IN PROGRESS...



$60 \leq \mathrm{N} < 90,\ 1 \leq h_i < \sqrt{2}$



N. Bilić, D.D. Dimitrijević, G.S. Djordjevic, M. Milošević & M. Stojanović,
Tachyon inflation in the holographic braneworld.
J. Cosmol. Astropart. Phys. 2019, 034–034 (2019).

CONCLUSIONS

- The ***simplest tachyon model*** that originates from the dynamics of a D3-brane in an AdS_5 bulk yields an inverse quartic potential.
- The same mechanism leads to a ***more general tachyon potential*** if the AdS_5 background metric is modified by the ***presence of matter in the bulk***, e.g. in the form of a minimally coupled scalar field with self-interaction.
- The software is written in such a way that its only inputs are the Hamilton's and Friedmann equations, with relevant parameters.
- The program can readily be used for a much wider set of models.
- The best fitting result is obtained for $V(\theta) = \frac{1}{\cosh \theta}$.
- Results indicate an opportunity for further research based on various potentials in the contest of the RSII model and holographic cosmology.

THANK YOU FOR YOUR ATTENTION

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