FROM STRING THEORY AND QUANTUM GRAVITY TO DARK MATTER AND DARK ENERGY

L. FREIDEL, R.G. LEIGH, D. MINIC → NEW QUANTUM FOUNDATIONS (1606.01829) (MODULAR SPACETIME BORN GEOMETRY)

(FLM) (1707.00312) ↕ METASTRING THEORY (1502.08005; 1706.03305) (⊕ J. KOWALSKI-GLIKMAN)

ALSO (1307.7080; 1405.3949)

↕ METAPARTICLES (1812.10821 ...)

C. M. HO, D. MINIC & Y. J. NG → DARK MATTER (THEORY)

(HMN) (1005.3537; 1105.2916; 1201.2635)

D. EDMONDS, D. FARRAH, C.M. HO, D. MINIC, Y.J. NG & T. TAKEUCHI

↳ DARK MATTER IN GALAXIES & G.CLUSTERS

(1308.3252; 1601.00662; 1709.04338) ...

D. EDMONDS, D. MINIC & T. TAKEUCHI → UNIVERSAL ACCELERATION IN GALAXIES, CLUSTERS (& COSMOLOGY) ...

(EMT)

P. BERGLUND, T. HÜBSCH, D. MINIC → DARK ENERGY (THEORY)

(BHM) (1902.08617; 1905.08269 1905.09463 ...)

2'

FLM

THE ESSENCE OF THIS TALK:

(RELATIVE LOCALITY; BORN GEOMETRY)

1) QUANTUM (MECHANICS) THEORY IN A MANIFESTLY NON-LOCAL (QUANTUM) FORMULATION VIA MODULAR VARIABLES & MODULAR (QUANTUM) SPACE-TIME GEOMETRY

(SCHWINGER; AHARONOV)

$\downarrow$

- PURELY QUANTUM MEASUREMENTS
- NEW CORRELATIONS / PHASES

CAUSALITY (BORN) (QUANTUM GEOMETRY)

$$\text{"}SO(D-1,1) = Sp(2D) \cap \underbrace{SO(D,D) \cap SO(2,2(D-1))}_{\text{NON-LOCALITY}}\text{"}$$

(QUANTUM)

2) QUANTUM FIELD THEORY IN A MANIFESTLY NON-LOCAL FORMULATION

$$\phi(x) \to \phi(x,\tilde{x}) \quad \& \quad [x,\tilde{x}] = 2\pi i \lambda^2$$

$$\left[\begin{array}{l} \omega \to Sp(2D) \\ \eta \to O(D,D) \\ H \to O(2,2(D-1)) \end{array}\right] \text{BORN GEOMETRY}$$

- NON-LOCAL QUANTA: "METAPARTICLES"
- NON-COMMUTATIVE STRUCTURE HIDDEN IN QFT

(NON-COMMUTATIVE GEOMETRY OF STANDARD MODEL) ← (CONNES) ⇝ (D.M. TAKEUCHI AYDEMIR SHAW) (EXACT)

? • (ORIGIN OF DUALITIES BTW QFTs; ORIGIN OF AMPLITUDES)

3) QUANTUM GRAVITY = "GRAVITIZING THE QUANTUM"

("STRING THEORY") METASTRINGS

(DYNAMICAL QUANTUM GEOMETRY)

$$\omega_{AB}(\mathbb{X}) = -\omega_{BA}(\mathbb{X}), \quad \eta_{AB}(\mathbb{X}) = \eta_{BA}(\mathbb{X}), \quad H_{AB}(\mathbb{X}) = H_{BA}(\mathbb{X})$$

$\Downarrow$

(DARK MATTER, DARK ENERGY)

HMN — BHM

EMT ..

(DEEP)

ANALOGY WITH RELATIVISTIC SPACE-TIME PHYSICS
(EINSTEIN)

1) SPECIAL RELATIVITY: MANIFEST COVARIANCE (POINCARÉ; LORENTZ;)
(ABSOLUTE CAUSALITY) FLAT SPACE-TIME GEOMETRY (MINKOWSKI)
RELATIVITY OF SPACE & TIME

2) CLASSICAL RELATIVISTIC FIELD THEORY: MANIFEST COVARIANCE
(LOCALITY ⊕ CAUSALITY) (REPS. OF LORENTZ GROUP)

3) GENERAL RELATIVITY: "GRAVITIZE SPECIAL RELATIVITY"
("LOCAL LORENTZ") (DYNAMICAL SPACE-TIME GEOMETRY)
BACKGROUND INDEPENDENCE (DYNAMICAL CAUSALITY)

IN WHAT FOLLOWS WE REVEAL THE QUANTUM GEOMETRY
BEHIND QUANTUM-NON-LOCALITY USING
(WEYL) SCHWINGER-AHARONOV'S MODULAR VARIABLES
AND THEN WE MAKE THIS QUANTUM GEOMETRY DYNAMICAL
(ABSOLUTE LOCALITY ⟶ RELATIVE LOCALITY)
LEADING TO QUANTUM GRAVITY ("STRING THEORY")
METASTRINGS
GENERIC PREDICTION: METAPARTICLES ⇒ (DARK MATTER & DARK ENERGY)

MODULAR VARIABLES & QUANTUM NON-LOCALITY

(WEYL; SCHWINGER
AHARONOV ET AL
FREIDEL, LEIGH, D.M.)

3'

INTERFERENCE WITHOUT ψ!

TWO-SLIT EXP

SCHRÖDINGER: $\psi(x) + e^{i\theta}\psi(x+R) \Rightarrow$ ESSENTIAL QUANTUM NON-LOCALITY

CLASSICAL (!) LABELS

AHARONOV'S QUESTION: WHAT QUANTUM OBSERVABLES CAPTURE THIS INTERFERENCE?

($[q,p] = i\hbar \Rightarrow$ NO POLYNOMIAL $P(\hat{q},\hat{p})$ WORKS!)

NEED SOMETHING THAT SHIFTS $x \to x+R \Rightarrow V(\hat{p}) = e^{\frac{i}{\hbar}\hat{p}R}$

$\Downarrow$

TRANSLATION (SHIFT) OPERATOR

(ALSO, $U(\hat{q}) = e^{i2\pi\hat{q}/R}$!)

CHECK: $[q,p] = i\hbar \Rightarrow UV = VU \Rightarrow [U,V] = 0$!!

COMMUTING QUANTUM OBSERVABLES (NO $\hbar \to 0$ LIMIT)

(NOTE: NO CLASSICAL ANALOG OF U & V !!)

AHARONOV'S INSIGHT: (U, V) OR MODULAR VARIABLES

$(p)_R \equiv \hat{p} \bmod \left(\frac{2\pi\hbar}{R}\right)$ & $(q)_R \equiv \hat{q} \bmod (R)$

CAPTURE THE ESSENCE OF QUANTUM THEORY: NON-LOCALITY THAT IS CONSISTENT WITH CAUSALITY !!

(SO "LOCAL" QFT IS POSSIBLE :))

MODULAR VARIABLES $(p)_R$, $(q)_R$ $\left([(q)_R, (p)_R] = 0 \, ! \right)$

APPEAR IN THE KINEMATIC NON-LOCALITY OF THE EPR-BELL TYPE

USE q & p (INSTEAD OF SPIN), AS IN THE ORIGINAL EPR PAPER,

$\Rightarrow$ $(p)_R$, $(q)_R$ NATURALLY APPEAR (ALSO GHZ)

(SEE ARXIV: QUANT-PH/0103048

OR ARXIV: 1212.5340)

BUT $(q)_R$ & $(p)_R$ CAPTURE DYNAMIC NON-LOCALITY :

LET $\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{q})$

"NON-LOCALITY IN YOUR FACE!"

$$\frac{d(p)_R}{dt} = -\left(\frac{V(q + \frac{R}{2}) - V(q - \frac{R}{2})}{R} \right)$$

→ BORN-HEISENBERG-JORDAN EOM

(ESSENTIALLY Q. EOM. FOR $e^{\frac{i}{\hbar}\hat{p}R}$!)

$\Downarrow$

WHERE $$\frac{d(p)_R}{dt} \equiv \frac{\hbar}{iR} \, e^{-\frac{i}{2\hbar}\hat{p}R} \left(\frac{d\, e^{\frac{i}{\hbar}\hat{p}R}}{dt} \right) e^{-\frac{i}{2\hbar}\hat{p}R}$$

EXAMPLES OF MODULAR VARIABLES:

1) AHARONOV-BOHM PHASE (CHARGED PROBE IN $\vec{B}$ FIELD)
2) AHARONOV-CASHER PHASE (SPIN PROBE IN $\vec{E}$ FIELD)

"FINAL" REMARKS: MODULAR VARIABLES ARE COMMUTING

THUS WE ~~NEED~~ TO LOOK AT COMMUTATIVE SUBALGEBRAS OF "$[\hat{q},\hat{p}]=i\hbar$" BORN-HEISENBERG-JORDAN

COMMUTATIVITY $\Rightarrow$ LATTICE IN PHASE SPACE

$\Leftarrow$ MODULAR SPACE

UNCERTAINTY PRINCIPLE: $\Leftarrow$ "BRILLOUIN" CELL

$\to$ CAN SPECIFY A POINT IN MODULAR CELL, BUT IF SO, CAN'T SAY WHICH CELL YOU ARE IN!!

FROM MODULAR POINT OF VIEW, THE SCHRÖDINGER PICTURE IS SINGULAR.

HOWEVER, ONE CAN UNITARILY MAP FROM MODULAR TO SCHRÖDINGER.

$\to$ THE MAP IS KNOWN IN SOLID STATE PHYSICS: ZAK TRANSFORM!

(USE $x=\frac{\hat{q}}{\lambda}$, $\tilde{x}=\frac{\hat{p}}{\epsilon}$, $[x,\tilde{x}]=\frac{1}{2\pi}$ $(\hbar=2\pi\lambda\epsilon)$)

MODULAR FUNCTIONS: $\Phi(x+n,\tilde{x}+\tilde{n}) = e^{2i\pi n\tilde{x}}\,\Phi(x,\tilde{x})$

MODULAR SCHRÖDINGER

ZAK: $\boxed{(Z_\lambda\varphi)(x,\tilde{x}) \equiv \sqrt{\lambda}\sum_{n\in\mathbb{Z}} e^{-2\pi i n\tilde{x}}\,\varphi(\lambda(x+n))}$

INVERSE ZAK: $\left[(Z_\lambda^{-1}\Phi)(x+n) = \frac{1}{\sqrt{\lambda}}\int_0^1 d\tilde{x}\, e^{2\pi i n\tilde{x}}\,\Phi(\lambda^{-1}x,\tilde{x})\right]$, $x\in[0,1)$

$\left(\hat{\tilde{x}} \to -\frac{i}{2\pi}\partial_x\,,\ \hat{x}\to\left(\frac{i}{2\pi}\partial_{\tilde{x}}+x\right)\right) \Rightarrow$ UNIT FLUX THROUGH MODULAR CELL!

FLM

QUANTUM THEORY IN TERMS OF MODULAR VARIABLES:

$$\hat{q} \to \hat{U} = \exp\left(\frac{2\pi i}{\hbar}\hat{q}\hbar R^{-1}\right) \equiv \exp\left(2\pi i \frac{\hat{q}}{R}\right)$$

$$\hat{p} \to \hat{V} = \exp\left(\frac{i}{\hbar}\hat{p}R\right)$$

"VERTEX OPERATORS"

$$\left([\hat{q}, \hat{p}] = i\hbar \;\Downarrow\; [\hat{U}, \hat{V}] = 0 \right)$$

USE $\hat{\mathbb{X}}^A = \begin{pmatrix} \hat{x}^a \\ \tilde{x}_a \end{pmatrix}$ $\quad \hat{x}^a \equiv \frac{\hat{q}^a}{\lambda}, \quad \hat{\tilde{x}}_a \equiv \frac{\hat{p}_a}{\epsilon} \quad (\hbar = \lambda\epsilon)$

$$\Downarrow$$

$$\underline{W_K \equiv \exp\left(2\pi i\, \omega(\hat{\mathbb{K}}, \hat{\mathbb{X}})\right)} \;;\; \underbrace{\omega(\mathbb{X}, \mathbb{Y}) = \hat{x}\cdot y - x\cdot\tilde{y}}_{Sp(2D)}$$

→ COMMUTATIVE SUBALGEBRA OF HEISENBERG ALGEBRA:

MODULAR SPACETIME $\Leftarrow$ SELF-DUAL LATTICE Λ WRT ω! (NARAIN LATTICE)

$$\Lambda = \ell \oplus \bar{\ell} \to \underbrace{\eta(\mathbb{P}, \mathbb{Q}) = \hat{p}\cdot q + p\cdot\hat{q}}_{O(D,D)} \;;\; \boxed{U_\lambda = e^{\frac{i}{2}\pi\eta(\lambda,\lambda)} W_\lambda}$$

FINALLY, VACUUM $\mathbb{P}^A|0\rangle = 0 \Rightarrow \underline{\hat{E}_H = H^{AB}\hat{\mathbb{P}}_A\hat{\mathbb{P}}_B}$, $\underline{\hat{E}_H|0\rangle = 0}$

(TRANSLATION GENERATOR, BUT TRANSLATIONS BROKEN)

(RELATIVE (OBSERVER DEPENDENT) LOCALITY)

$\underline{H^{AB} = H^{BA}}$ $\quad \underline{O(2, 2(D-1))}$

NOTE: LORENTZ!

$$\boxed{O(D-1,1) = Sp(2D) \cap O(D,D) \cap O(2, 2(D-1))}$$

$$\underbrace{\omega_{AB} \quad \eta_{AB} \quad H_{AB}}_{\text{BORN GEOMETRY}}$$

FLM

SUMMARY:

(SCHWINGER; AHARONOV)

MODULAR VARIABLES: FROM QUANTUM THEORY TO STRING THEORY (QUANTUM GRAVITY)

I) QUANTUM THEORY (MECHANICS): NON-LOCALITY ⊕ CAUSALITY $R \updownarrow$

(AHARONOV)

INSTEAD OF $\hat{q}, \hat{p}$ WITH $[\hat{q}, \hat{p}] = i\hbar$ USE $[\hat{q}]_R \equiv \hat{q} \bmod (R)$

$[\hat{p}]_R \equiv \hat{p} \bmod \left(\frac{2\pi\hbar}{R}\right)$

(NON-LOCAL MODULAR VARIABLES $[\hat{q}]_R$, $[\hat{p}]_R$) WITH $[[\hat{q}]_R, [\hat{p}]_R] = 0$!

FULLY COVARIANT ! $\Downarrow$ MODULAR SPACETIME

II) (LOCAL) QUANTUM FIELD THEORY POSSIBLE BECAUSE OF CONSISTENCY BTW QUANTUM NON-LOCALITY & CAUSALITY

→ • METAPARTICLES !

(STILL NEW INSIGHTS AND PHYSICS! PROVIDED BY MODULAR VARIABLES)

("NON-LOCALITY" ... "WEAK MEASUREMENTS" ETC)

$\Uparrow \quad \Downarrow$

III) (QUANTUM GRAVITY) STRING THEORY IN METASTRING FORMULATION

- LIVES IN MODULAR SPACE TIME
- "GRAVITIZES" THE GEOMETRY OF QUANTUM THEORY IN MODULAR FORMULATION

$(\omega_{AB}(\mathbb{X}),\ \eta_{AB}(\mathbb{X}),\ H_{AB}(\mathbb{X}))$

FLM

- R is λ !

CONTEXTUAL

NOT-CONTEXTUAL IN QUANTUM GRAVITY

(B)

(T-DUALITY) METASTRING THEORY FLM

COVARIANT FORMULATION (METASTRING) ("PHASE SPACE") $\longleftrightarrow \mathbb{X}^A = \begin{pmatrix} x^a/\lambda \\ \tilde{x}_a/\epsilon \end{pmatrix}$

$$S_{MS} = \frac{1}{4\pi}\int \left[\partial_\tau \mathbb{X}^A (\eta_{AB} + \omega_{AB})(\mathbb{X})\, \partial_\sigma \mathbb{X}^B - \partial_\sigma \mathbb{X}^A H_{AB}(\mathbb{X})\, \partial_\sigma \mathbb{X}^B \right]$$

$\left(\hbar = \lambda\epsilon \quad \alpha' = \frac{\lambda}{\epsilon} \right)$

$\begin{pmatrix} 0 & \delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix} \leftarrow O(D,D)$ $\quad Sp(2D) \hookrightarrow \begin{pmatrix} 0 & -\delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix}$ $\quad O(2,2(D-1)) \left[\begin{pmatrix} G_{ab} & 0 \\ 0 & G_{ab}^{-1} \end{pmatrix} \right]$

THEN IMMEDIATELY:

$$\boxed{[\hat{\mathbb{X}}^A, \hat{\mathbb{X}}^B] = 2i\left[\pi\omega^{AB} - \eta^{AB}\, \theta(\sigma,\sigma')\right]}$$

"STAIRCASE DISTRIBUTION"

NOTE $\quad J \equiv \eta^{-1} H, \quad J^2 = 1$

$\left(\mathbb{X}^A(\sigma + 2\pi) = \mathbb{X}^A(\sigma) + \Delta^A \right)$

GENERAL MONODROMIES

T-DUALITY $\quad \mathbb{X} \to J(\mathbb{X})$

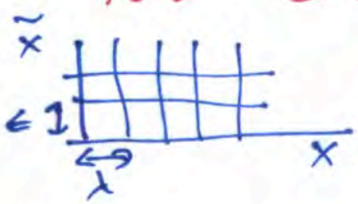
CHIRAL FORMULATION (AND NON-LOCAL) $\longmapsto$ "FULLY COMPACTIFIED" $\leftarrow$ MONSTER, BORCHERDS.

D = 26 BOSONIC STRING $\longleftrightarrow$ (UNIQUE SELF-DUAL LORENTZIAN LATTICE)

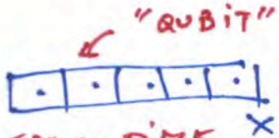
No CTCs $\rightarrow$ "MODULAR SPACETIME" (QUANTUM)

$\Downarrow$

MODULAR VARIABLES OF QUANTUM THEORY



$\Rightarrow$



SPACETIME "EXTENSIFICATION"

EFFECTIVE PICTURE OF STRINGS:

"TARGET SPACE = INDEX SPACE"

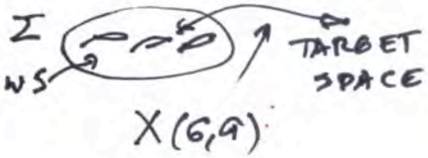
- OLD PICTURE:

$\xrightarrow{L \gg \lambda}$

"PARTICLES"

INDEX SPACE

$\Phi(x)$ – LOCAL FIELD



(EFFECTIVE FIELD THEORY / WILSONIAN RG

PARTICLE QUANTA

DERIVATIVE EXPANSION

DECOUPLING OF IR & UV)

...

- NEW PICTURE:

FLM

$\xrightarrow{L \gg \lambda}$

"METAPARTICLES"

$[x, \tilde{x}] = i\lambda^2 (2\pi)$

$\Phi(x, \tilde{x})$ – NON-COMMUTATIVE FIELD THEORY

$(\hbar = \lambda\epsilon$

$\alpha' = \frac{\lambda}{\epsilon})$

(VIOLATION OF DECOUPLING

UV/IR MIXING

DOUBLE SCALE RG; SELF-DUAL FIXED POINT)

$\boxed{\text{"TARGET SPACE} \neq \text{INDEX SPACE"}}$

CONSIDER

FLAT (FREE) STRING: $S_P \sim \int {}^*dx^\mu \wedge dx^\nu \, h_{\mu\nu}$

$\delta S_P = 0 \longrightarrow \square X = 0 \rightarrow$ 2 SOLUTIONS

$$\boxed{X(\tau,\sigma) = X_R(\tau+\sigma) + X_L(\tau-\sigma),} \qquad \lambda \equiv \sqrt{\frac{\pi\alpha'}{2}}$$

$$X_L(\tau-\sigma) \equiv x_L + \frac{\alpha'}{2} p_L (\tau-\sigma) + i\lambda \sum_{n=-\infty}^{+\infty} \frac{1}{n} \alpha_n e^{-in(\tau-\sigma)}$$

$$X_R(\tau+\sigma) \equiv x_R + \frac{\alpha'}{2} p_R (\tau+\sigma) + i\lambda \sum_{n=-\infty}^{+\infty} \frac{1}{n} \tilde{\alpha}_n e^{-in(\tau+\sigma)}$$

ALSO:

$$\boxed{\tilde{X}(\tau,\sigma) = X_R(\tau+\sigma) - X_L(\tau-\sigma)}$$

NOTE: $dX = {}^*d\tilde{X}$, i.e. $\partial_\tau X = \partial_\sigma \tilde{X}$; $\partial_\sigma X = \partial_\tau \tilde{X}$

STRINGS ON S^1:

$\tilde{X} \rightarrow$ T-DUALITY $\qquad \tilde{X}(\tau, \sigma+2\pi) = \tilde{X}(\tau,\sigma) + 2\pi\alpha' p \qquad \left(p = \frac{p_L + p_R}{2}\right)$

$(R; \bigcirc \rightarrow \bigcirc; \tilde{R} \;;\; R\tilde{R} = 2\lambda^2) \qquad \left(2\pi\alpha' p = \int_0^{2\pi} d\sigma \, \partial_\tau X\right)$

CIRCLE $\qquad$ "DUAL" CIRCLE

USUALLY $[x, \tilde{x}] = 0$ (TEXTBOOK)

"ZERO MODES x_L, x_R COMMUTE"

$\boxed{\text{NOT TRUE!}}$

4 DIFFERENT CALCULATIONS TO SEE THAT $[x, \tilde{x}] = i\lambda^2 (2\pi)$

1) PERHAPS THE SIMPLEST: START FROM CANONICAL COMM.

$$[\hat{X}(\tau, \sigma_1), \partial_\tau \hat{X}(\tau, \sigma_2)] = 2\pi i \hbar \alpha' \delta(\sigma_{12})$$

HOWEVER, $\partial_\tau X = \partial_\sigma \tilde{X}$

$$[\hat{X}(\tau, \sigma_1), \partial_\sigma \hat{\tilde{X}}(\tau, \sigma_2)] = 2\pi i \hbar \alpha' \delta(\sigma_{12})$$

INTEGRATE OVER σ; INVOKE (WORLD-SHEET) WS CAUSALITY (MUTUAL LOCALITY)

FLM $\quad [\hat{X}(\tau, \sigma_1), \hat{\tilde{X}}(\tau, \sigma_2)] = 2\pi i \lambda^2 [\pi - \theta(\sigma_{12})]$ STAIRCASE DISTRIBUTION

ZERO MODES DO NOT COMMUTE $\quad [\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b$

$(x \equiv x_L + x_R; \ \tilde{x} \equiv x_L - x_R)$

HEISENBERG ALGEBRA

USE THE DOUBLE NOTATION:

$$\mathbb{X}^A(\tau,\sigma) \equiv \left(X^\mu(\tau,\sigma),\ \tilde{X}_\mu(\tau,\sigma) \right)$$

$$\Rightarrow \left[\hat{\mathbb{X}}^A(\tau,\sigma_1),\ \hat{\mathbb{X}}^B(\tau,\sigma_2) \right] = 2i\lambda^2 \left[\pi\omega^{AB} - \eta^{AB}\theta(\sigma_{12}) \right]$$

ZERO MODES

OSCILLATORS

$\omega^{AB} \to Sp(2D)$

$\hookrightarrow$ dim OF SPACETIME

$\eta^{AB} \to O(D,D)$

$\theta(\sigma)$ - STAIRCASE DISTRIBUTION

2) WHERE IS $\tilde{X}$ IN THE POLYAKOV ACTION?

T-DUALITY $\to$ $S_P \to S_P + \hbar \int \omega$ $\Big] \Rightarrow$ METASTRING S_{MS}

RLM

$$\omega \equiv \frac{1}{8\pi\lambda^2} \left[d\tilde{X}_a \wedge dX^a \right]$$

WZW - LIKE TERM

$\tilde{X}$ - TOPOLOGICAL DEGREE OF FREEDOM

(ONLY ZERO MODES DYNAMICAL)

3) SYMPLECTIC FORM FROM δS_P

$$\left(S_P \equiv \frac{1}{4\pi\alpha'} \int d^2\sigma \left[(\partial_\tau X)^2 - (\partial_\sigma X)^2 \right] \right)$$

BE CAREFUL WITH BOUNDARY TERMS

FOR A CYLINDRICAL $\underline{WS}$ CUT OPEN AT $\sigma = 0$

$$(\sigma \in [0, 2\pi], \quad \tau \in [\tau_0, \tau_1])$$

USE $\partial_\tau X = \partial_\sigma \tilde{X}$ ON THE BOUNDARY

$\delta S_P \rightarrow$ SYMPLECTIC 1-FORM $\Theta(\tau)$

$\delta\Theta = \Omega$ - SYMPLECTIC 2-FORM

FLM

$$\rightarrow \boxed{\Omega = \delta p \wedge \delta x + \delta\hat{p} \wedge (\delta\tilde{x} - \pi\alpha'\delta p) + \text{OSCILLATOR TERMS}}$$

BERRY-PHASE-LIKE TERM : $-\pi\alpha'\delta\hat{p} \wedge \delta p$

(METAPARTICLE)

$$\Rightarrow \underline{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$$

(+) CANONICAL COMMUTATORS

4) COCYCLES OF TEXTBOOK VERTEX OPERATORS:

$$\hat{U}_k = e^{ik_L x_L + ik_R x_R}$$ (COMMUTING ZERO MODES ! WITH)

$$\hat{U}_k \hat{U}_{k'} = \epsilon_{k,k'} \hat{U}_{k+k'}$$

DEDUCE (FROM MUTUAL LOCALITY) $\epsilon_{k,k'} = e^{2\pi i \lambda^2 \tilde{k} k'}$

REALIZE INTRINSIC NONCOMMUTATIVITY $[x, \tilde{x}] = 2\pi i \lambda^2$

NOW $\hat{U}_k = e^{ik\hat{x}} e^{i\tilde{k}\hat{\tilde{x}}}$ $\quad (x = x_L + x_R)$ $\quad (\tilde{x} = x_L - x_R)$

$\downarrow$

REPS OF HEISENBERG-WEYL

COCYCLES $\rightarrow$ 1 !

FLM

(NEW INSIGHT ON ASYMMETRIC ORBIFOLDS & OTHER NON-GEOMETRIC BACKGROUNDS)

FLM

TURN ON $\underset{\substack{\| \\ G_{ba}}}{G_{ab}}$ & $\underset{\substack{\| \\ -B_{ba}}}{B_{ab}}$:

$$S_{P(G,B)} = \frac{1}{4\pi\alpha'} \int d^2\sigma \left(G_{ab} \left[\partial_\tau X^a \partial_\tau X^b - \partial_\sigma X^a \partial_\sigma X^b \right] + 2 B_{ab} \partial_\tau X^a \partial_\sigma X^b \right)$$

$$\Rightarrow \quad \boxed{[\hat{x}^a, \hat{x}^b] = 0}, \quad \boxed{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}, \quad \boxed{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = -4\pi i \lambda^2 B_{ab}}$$

INTRODUCE $\underset{O(D,D)}{\eta_{AB}} = \begin{pmatrix} 0 & \delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}$ $\quad \underset{Sp(2D)}{\omega_{AB}} = \begin{pmatrix} 0 & -\delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}$ $\quad \boxed{\begin{matrix} K = \eta^{-1}\omega \\ K^2 = 1 \end{matrix}}$

IF $B_{ab} = 0$ $\quad \boxed{\Omega = \eta_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{X}^B + \frac{\pi\alpha'}{2} \omega_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{P}^B}$

IF $B_{ab} \neq 0$ $\quad \eta^{(B)}_{AB} = \eta_{AB} = \begin{pmatrix} 0 & \delta_a{}^b \\ \delta^a{}_b & 0 \end{pmatrix}, \quad \omega^{(B)}_{AB} = \begin{pmatrix} -2B_{ab} & -\delta_a{}^b \\ \delta_a{}^b & 0 \end{pmatrix}$

$$\boxed{\Omega = \eta_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{X}^B + \frac{\pi\alpha'}{2} \omega^{(B)}_{AB}\, \delta \mathbb{P}^A \wedge \delta \mathbb{P}^B}$$

THEN $\boxed{[\mathbb{X}^A, \mathbb{X}^B] = 2\pi i \lambda^2 (\omega^{(B)})^{AB}}$, $\quad (\omega^{(B)})^{AB} = \begin{pmatrix} 0 & \delta^a{}_b \\ -\delta_a{}^b & \underbrace{-2B_{ab}} \end{pmatrix}$

!!

B_{ab} is in ω_{AB}

FLM

(9)

NOTE: B-FIELD TRANSFORMATION

$$\mathbb{X} = (x^a, \tilde{x}_a) \overset{B}{\mapsto} (x^a, \tilde{x}_a + B_{ab}x^b)$$

WHICH LEADS TO $\underbrace{[\hat{x}^a, \hat{x}^b] = 0}$, $\underbrace{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}$, $\underbrace{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = -4\pi i \lambda^2 B_{ab}}$

HOWEVER, WE CAN ALSO CONSIDER

$$(x^a, \tilde{x}_a) \overset{\beta}{\longmapsto} (x^a + \beta^{ab}\tilde{x}_b, \tilde{x}_a)$$

"STRINGY" (NON-LOCALITY)

THIS LEADS TO NON-COMMUTATIVITY OF x^a!

$$\underbrace{[\hat{x}^a, \hat{x}^b] = 4\pi i \lambda^2 \beta^{ab}}, \underbrace{[\hat{x}^a, \hat{\tilde{x}}_b] = 2\pi i \lambda^2 \delta^a_b}, \underbrace{[\hat{\tilde{x}}_a, \hat{\tilde{x}}_b] = 0}$$

FINALLY, NOTE THAT FOR THE B-FIELD BACKGROUND

$$\underline{[\hat{\tilde{x}}_a, [\hat{\tilde{x}}_b, \hat{\tilde{x}}_c]] + \text{cyclic} = H_{abc}(x)}$$

$H_{abc} = \partial_a B_{bc} + \text{cyclic}$ $\longrightarrow$ H-FLUX $\longleftrightarrow$ $\boxed{\text{"MONOPOLE"}}$ 3-COCYCLES

POSSIBILITY OF NON-ASSOCIATIVITY.

ALL FLUXES $(H, \underbrace{F, R}_{\text{DOUBLE FIELD THEORY}} \ldots)$ FROM NON-COMMUTATIVITY & NON-ASSOCIATIVITY OF "PHASE SPACE" $\mathbb{X}$

FLM

EFFECTIVE FIELD DESCRIPTION OF CLOSED STRINGS

$$\boxed{\Phi(x,\hat{x}) \equiv \sum_{w} \phi_w(x)\, e^{iw\tilde{x}/\hat{\ell}}}$$

$$[x,\hat{x}] = 2\pi i \lambda^2$$

THUS

$$\Phi(x,\tilde{x}) = \Phi(x) + \overbrace{\sum_{w\neq 0} \phi_w(x)\, e^{iw\tilde{x}}}^{\text{NON-LOCAL FIELD}}$$

(DOUBLE RG
UV/IR MIXING)
SELF-DUAL FIXED POINTS

LOCAL EFFECTIVE FIELD

NON-PARTICLE QUANTA OF $\Phi(x,\tilde{x})$:

FROM NON-COMMUTATIVE CLOSED STRING ZERO MODES

"METAPARTICLES"

FLM
⊕
KOWALSKI-GLIKMAN

$$S_{MP} = \int \Big[p\dot{x} + \hat{p}\dot{\hat{x}} - \alpha' \pi \underbrace{p\hat{p}}_{\omega} + N_1 (\overbrace{p^2+\hat{p}^2}^{H} - M_1^2) + N_2 (\underbrace{p\hat{p}}_{\eta} - M_2^2) \Big]$$

(FK-GLM) CONSTRAINTS $(p^2+\hat{p}^2 = M_1^2)$ & $(p\hat{p} = M_2^2)$

FROM METASTRING CONSTRAINTS

$$\Rightarrow \begin{bmatrix} \underline{H}: \partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^B H_{AB} \approx 0 \\ \underline{D}: \partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^B \eta_{AB} \simeq 0 \end{bmatrix}$$

(USUALLY
WE IGNORE $\tilde{x}$!!
"$(\tilde{x}\to 0)$"
OR $\underline{(\hat{p}\to 0)}$ & $(M_2\to 0)$)

STRING INTERACTIONS :

INCORPORATE
$$[x, \tilde{x}] = 2\pi i \lambda^2$$

$(\Phi \circ \psi)_w(x)$

$\Phi_{w'}(x)$ $\qquad$ $\psi_{w-w'}(x)$

FLM

$$\Phi_w(x) = \Phi^{(+)}(x, x+\pi w R)\, \Phi^{(-)}(x+\pi w R, x+2\pi w R)$$

CLOSED STRING ~ PRODUCT OF TWO OPEN STRINGS

$$(\Phi \circ \psi)^{(\pm)}_{w'}(x, x+\pi w R) \equiv \Phi^{(\pm)}(x, x+\pi w' R)\, \psi^{(\pm)}(x+\pi w' R, x+\pi w R)$$

REALIZATION OF THE HEISENBERG GROUP $[x, \tilde{x}] = 2\pi i \lambda^2$

?! NON-PERTURBATIVE FORMULATION $\quad \partial_\sigma \hat{\mathbb{X}}^A \to \frac{1}{\lambda}[\hat{\mathbb{X}}^{26}, \hat{\mathbb{X}}^A]$

INVOLVING $\hat{\mathbb{X}}^A(\sigma, \tau) \to \hat{\mathbb{X}}^a(\tau)$ $\quad A = 0, 1 \ldots 25 \longrightarrow a = 0, 1, \ldots, 25, 26$

$$S_{NP} \sim \frac{1}{\lambda}\int \mathrm{Tr}\left(\partial_\tau \hat{\mathbb{X}}^a [\hat{\mathbb{X}}^b, \hat{\mathbb{X}}^c]\, \eta_{abc}^{(*)} - \frac{1}{\lambda} H_{ac}^{(*)} [\hat{\mathbb{X}}^a, \hat{\mathbb{X}}^b][\hat{\mathbb{X}}^c, \hat{\mathbb{X}}^d] H_{bd}^{(*)} \right)$$

"$27 = 11 + 16$" $\;\hookrightarrow$ SUSY, EMERGENT

$\downarrow$

M-THEORY

11D – M-THEORY

COVARIANT MATRIX THEORY

$$(\eta_{abc}^{(*)} \to \omega_{AB}^{(*)} + \eta_{AB}^{(*)})$$

FLM = { FREIDEL, LEIGH, MINIC }

1307.7080; 1405.3949
1502.08005; 1606.01829
1706.03305; 1707.00312
ETC

METAPARTICLES

FLM ⊕ KOWALSKI – GLIKMAN
1812.10821

①

USUAL RELATIVISTIC PARTICLE

$$S_p = \int p\dot{x} - N(\underbrace{p^2+m^2}_{H})$$ → CLASSICAL ACTION

$$\left[\begin{array}{c} H = p^2+m^2 \\ H \approx 0 \end{array}\right]$$

$H\phi = 0$

QUANTUM PROPAGATOR:

$$\underset{\underset{\langle x|0\rangle}{|||}}{G(x,0)} \sim \int \frac{d^Dp}{(2\pi)^D}\int_0^\infty dN\, e^{-iNH}\, e^{ipx} = \int\frac{d^Dp}{(2\pi)^D}\frac{e^{ipx}}{p^2+m^2-i\epsilon}$$

FEYNMAN $$\frac{1}{p^2+m^2-i\underline{\epsilon}}$$ ← CAUSALITY

QUANTUM FIELD $\phi(x)$

$$S_\phi = \int \phi(\Box+m^2)\phi + \overset{\text{INTERACTIONS}}{(\phi^3\dots)}$$

QUANTUM FIELD THEORY (QFT)

$$\int D\phi\, e^{iS_\phi + \underset{\text{SOURCES}}{J\phi}}$$

'FIRST QUANTIZED'

$$\sum_{\text{topology}}\int Dx\,Dp\, e^{iS_p}$$

"SECOND QUANTIZED" ϕ^3

$$\int D\phi\, e^{iS_\phi}$$

OR $\hat{\phi}(x)$ → "POINT PARTICLE" AT x

↑ (BOTH PARTICLE & ANTIPARTICLE)
QUANTUM FIELD
(OPERATOR)

↑ CLASSICAL LABEL
CLASSICAL (SPACETIME)

QFT = CLASSICAL RELATIVITY ⊕ QUANTUM MECHANICS/THEORY

1) SPECIAL RELATIVITY: CLASSICAL SPACETIME × [RELATIVISTIC KINEMATICS $p^2+m^2=0$, $E=\pm\sqrt{\vec{p}^2+m^2}$ (± PARTICLE & ANTIPARTICLE)]

FIXED MINKOWSKI GEOMETRY; LORENTZ

2) RELATIVISTIC FIELD: REPS. OF LORENTZ GROUP $SO(\overbrace{D-1,1}^{D})$

(S_ϕ) ⟷ $\phi(x)$ ⟶ [IN QUANTUM THEORY $\hat{\phi}(x)$ (OPERATOR)]

↑ CLASSICAL SPACETIME

[3) GENERAL RELATIVITY: DYNAMICAL SPACETIME GEOMETRY $(g_{\mu\nu})$

$(S(g;\phi))$

$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R \equiv G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$

$$S=\int\sqrt{-g}\,R+\int\sqrt{-g}\,L_m$$

$\left(T_{\mu\nu}=\frac{\partial L_m}{\partial g^{\mu\nu}}-\frac{1}{2}L_m g_{\mu\nu}\right)$

$$\left[T_{\mu\nu}=\underbrace{\lambda g_{\mu\nu}}_{\text{DARK ENERGY}}+\underbrace{T_{\mu\nu}^{(DM)}}_{\text{DARK MATTER}}+\underbrace{T_{\mu\nu}^{(SM)}}_{\text{STANDARD MODEL (BARYONIC) MATTER}}\right]$$

QUANTUM RELATIVITY (FLM)

I) QUANTUM MECHANICS: "MODULAR" QUANTUM SPACETIME $\begin{pmatrix}x^a\\ \tilde{x}_a\end{pmatrix}\equiv \mathbb{X}^A$ $\left([x,\tilde{x}]=i\lambda^2\right)$

||| QUANTUM ("BORN") GEOMETRY

(1) NON-LOCALITY ⊕ CAUSALITY $(SO(D))$ {FIXED QUANTUM ("BORN") GEOMETRY

$$SO(D)=\underset{\omega_{AB}}{Sp(2D)}\cap \underset{\eta_{AB}}{SO(D,D)}\cap \underset{H_{AB}}{SO(2,2(D-1))}$$

λ - FUNDAMENTAL LENGTH/TIME

II) QUANTUM FIELD THEORY: $\underbrace{\Phi(\hat{x},\hat{\tilde{x}})}_{\text{METAPARTICLE !}}$, $[\hat{x}^a,\hat{\tilde{x}}_b]=i\lambda^2\delta^a_b$ (OR $[\hat{\mathbb{X}}^A,\hat{\mathbb{X}}^B]=i\omega^{AB}$)

[III) QUANTUM GRAVITY: GENERAL QUANTUM RELATIVITY ("GRAVITIZE THE QUANTUM")

$\omega(\mathbb{X})$, $\eta(\mathbb{X})$, $H(\mathbb{X})$

"UNIQUE" QUANTUM THEORY

$$S_{MS}=\int \partial_a\mathbb{X}^A\left(\omega_{AB}^{(\mathbb{X})}+\eta_{AB}^{(\mathbb{X})}\right)\partial_b\mathbb{X}^B-\partial_b\mathbb{X}^A H_{AB}^{(\mathbb{X})}\partial_b\mathbb{X}^B$$

METASTRING $\Rightarrow [\mathbb{X}^A,\mathbb{X}^B]=i\omega^{AB}$ (USUAL STRING $\omega\to 0$, $\eta\to 1$)]

METAPARTICLE : $\left[S_{MP} = \int [\, p\dot{x} + \tilde{p}\dot{\tilde{x}} - \underbrace{\lambda^2 p\hat{p}}_{\omega_{AB}} + N(\overbrace{p^2 + \hat{p}^2}^{H_{AB}} + M_1^2) + \tilde{N}(\underbrace{p\tilde{p}}_{\eta_{AB}} + \hat{M}_2^2) \,] \right.$

$(\tilde{p} \to 0 \Rightarrow S_{MP} \equiv S_P$

"METASTRING ZERO MODE"

$\downarrow$

GENERIC QUANTUM PARTICLE : QUANTUM OF $\phi(x, \tilde{x})$ $[x, \tilde{x}] = i\lambda^2$

METAPARTICLE FIELD

3

(TAKE ANY STANDARD MODEL FIELD $\phi(x) \longrightarrow \phi(x, \hat{x})$)

THUS ANY STANDARD MODEL PARTICLE $\Rightarrow$ METAPARTICLE)

NOTE : TWO CONSTRAINTS

$H \equiv p^2 + \tilde{p}^2 + M_1^2 \simeq 0$

$D \equiv p\hat{p} + M_2^2 \simeq 0$

DERIVED!

$(p \sim 10^{19}\,\text{GeV}$ & $\tilde{p} \sim 10^{-3}\,\text{eV})$

$\Rightarrow M_1 \sim 10^{19}\,\text{GeV}$ & $M_2 \sim 1\,\text{TeV}$ $\to$ MIXING OF UV & IR SCALES!

(UNITARY & CAUSAL)

QUANTUM PROPAGATOR:

$$G(x, \tilde{p}; 0, \tilde{p}_i) \sim \delta^{(D)}(\hat{p} - \tilde{p}_i) \int \frac{d^D p}{(2\pi)^D} \frac{e^{ipx}}{p^2 + \hat{p}^2 + M_1^2 - i\epsilon} \delta(p\hat{p} + M_2^2)$$

QUANTUM (METAPARTICLE) FIELD φ $\quad S = \int \varphi H \varphi + I(P_D \varphi)$ $\leftarrow$ interactions

$(P_D^2 = P_D) \leftarrow P_D$ - PROJECTOR ON THE KERNEL OF D (i.e. $D\varphi = 0$)

"MODES NOT IN THE KERNEL OF D DECOUPLE"

$$G = \frac{P}{H + i\epsilon}$$

- METAPARTICLE PROPAGATOR

("DUAL GAUGE THEORY")

STATIC POTENTIAL :

D - SPACETIME DIMENSIONS

STANDARD PARTICLE : $V(r) \sim \int d^{D-1}p \, \frac{e^{ipr}}{p^2+m^2} \sim \frac{1}{r^{D-3}} e^{-mr}$

(YUKAWA)

(4)

METAPARTICLE:

$$V(r, \tilde{r}) \sim \int d^{D-1}p \, d^{D-1}\tilde{p} \, e^{ipr} e^{i\tilde{p}\tilde{r}} \frac{\delta(p\tilde{p}-\mu)}{p^2+\tilde{p}^2+m^2}$$

$$V(r, \tilde{r}) \rightsquigarrow \frac{1}{r^{2(D-3)}} e^{-\left(\sqrt{\frac{\mu}{2}+m^2}\, r\right)} \sin\left(\sqrt{\frac{\mu}{2}-m^2}\, r\right) + \ldots$$

$\left(r_{\substack{L,R \\ (+),(-)}} = r \pm \Delta \right)$

$\left(r_{\pm} = r \pm \tilde{r} \right)$

IN $D=4$

$$\boxed{V(r_+, r_-) \sim \frac{1}{r_+ r_-} \underbrace{e^{-\left(\sqrt{\frac{\mu}{2}+m^2}\, r_-\right)}}_{\text{YUKAWA}} \underbrace{\sin\left(\sqrt{\frac{\mu}{2}-m^2}\, r_+\right)}}$$

"FRIEDEL OSCILLATION"

$\Downarrow$

QUANTUM

IN CONDENSED MATTER PHYSICS

FRIEDEL OSCILLATIONS $\left(V(r) \sim \frac{1}{r^3} \sin(2k_F r) \right)$

MEASURED

FERMI MOMENTUM

PHENOMENOLOGY OF METAPARTICLES:

GENERIC PREDICTION OF QUANTUM THEORY IN MANIFESTLY NON-LOCAL (& CAUSAL) FORM

5

1) EACH PARTICLE OF THE STANDARD MODEL HAS A "DUAL" PARTICLE

$\hat{\phi}(x)$ ⇒ PARTICLE E ⟺ ANTIPARTICLE $-E$

$\hat{\phi}(x,\tilde{x})$:

DUAL PARTICLE $\frac{M_2^2}{E}$ ⟺ DUAL ANTIPARTICLE $-\frac{M_2^2}{E}$

DUAL PARTICLES → DUAL STANDARD MODEL (HIDDEN SECTOR?)

2) A PARTICLE (PHOTON, NEUTRINO, QUARK...) ENTANGLED WITH ITS DUAL

DUAL PHOTON DUAL NEUTRINO DUAL QUARK...

$$p^2 + \tilde{p}^2 + M_1^2 = 0 \quad \& \quad p\tilde{p} = M_2^2$$

ALSO "BERRY PHASE IN MOMENTUM SPACE" $-\lambda^2 p\tilde{p}$ in S_{MP}

3) METAPARTICLES $\phi(x,\tilde{x})$:⇒ DARK MATTER QUANTA CORRELATED TO VISIBLE (BARYONIC/STANDARD MODEL) QUANTA

(WITH DOUGE, TATSUTET AL) (EDMONDS) (TAKEUCHI) (HO, FARRAR, NG)

$$M_{DM} = M_B\, f\left(\frac{a}{a_0}, \frac{r}{r_c} \cdots\right)$$

$a_0 \sim 10^{-10}\,\frac{m}{s^2} \to 10^3\,eV$ (DARK ENERGY)

4) GEOMETRY OF $\tilde{X}$:⇒ DARK ENERGY (WITH BERGLUND & HUBSCH)

$$\int \sqrt{-g(x)}\,\sqrt{-\tilde{g}(\tilde{x})}\,\left(R(x) + \tilde{R}(\tilde{x}) + \ldots\right) \to \frac{1}{G_N}\int \sqrt{g}\,R + \Lambda \int \sqrt{-g(x)}$$

$\frac{1}{G_N}$: $\left(\int \sqrt{-\tilde{g}(\tilde{x})}\right)$

Λ: $\left(\int \sqrt{-\tilde{g}(\tilde{x})}\,\tilde{R}(\tilde{x})\right)$

(DARK ENERGY) COSMOLOGICAL CONSTANT

BHM

COMMENTS:

BHM

1) DARK ENERGY $\Lambda \sim \int \sqrt{-\tilde{g}(\tilde{x})}\, \tilde{R}(\tilde{x})$ $\left(M_\Lambda \sim \dfrac{M^2}{M_P} \right)$ $\uparrow$ $10^{-3}\,eV$ if $M \sim 1\,TeV$

$\Downarrow$ $M \sim M_P e^{-\# \frac{M_P}{M_c}}$

$\Lambda \sim M_P^4 \exp\left(-\# \dfrac{M_P}{M_c}\right)$

RADIATIVELY STABLE

$\Downarrow$

SEQUESTERED EFFECTIVE FIELD THEORY

$\Downarrow$ (KALOPER, PADILLA...)

(6)

NON-PERTURBATIVE STABILITY $\rightarrow$ NON-COMMUTATIVE FIELD THEORY

(INFLATION $\rightarrow$ HIGHER ORDER R^2

H_0 TENSION $\rightarrow$ HIGHER ORDER $\hat{R}^2$)

(UV/IR ; DOUBLE-SCALE RG SELF-DUAL FIXED POINTS)

2) DARK MATTER $\phi(x, \tilde{x})$ $[x, \tilde{x}] = i\lambda^2$

HMN SENSITIVE TO Λ (DARK ENERGY)

EMT (W/EDMONDS, TAKEUCHI ; W/E, T, HO, NG, FARRAH)

$a_0 \leftrightarrow M_\Lambda = \dfrac{M^2}{M_P}$

$(10^{-10}\frac{m}{s^2}) \leftrightarrow (10^{-3}\,eV) = \dfrac{(1\,TeV)^2}{(10^{19}\,GeV)}$

$\int \sqrt{-g\tilde{g}} \left[\mathcal{L}_m(x) + \mathcal{L}_m(\tilde{x}) \right]$

$\wr\wr$

$\underbrace{\tilde{\mathcal{L}}_m(x)}$]

DARK MATTER SENSITIVE TO Λ AND $\mathcal{L}_m(x)$

$\rightarrow$ HIERARCHICAL STRUCTURE FORMATION (FROM QUANTUM GRAVITY)